Towards understanding black hole non-equilibrium thermodynamics

Daichi Takeda (Kyoto U)
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Self introduction

Name
Daichi Takeda

Status
PhD student (final year from next April)

Field
High energy physics

Research topics
String field theory

Holography

Black hole thermodynamics

Searching QG macroscopically

Origin of spacetime is unknown

Quantum theory of gravity is necessary

On the other hand, BH is thermodynamic

Thermodynamics is macroscopic

BH thermodynamics will give macroscopic clues to QG!

A formulation of black hole thermodynamics out of equilibrium guided by holography

1. BH thermodynamics in equilibrium
2. Problem: 2nd law of BH thermodynamics
3. Holography: duality between gravity and QFT
4. What holography tells about BHT?

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4 laws in thermodynamics

0th law: Existence of intensive variables

Temperature T, chemical potential μ

1st law: Energy conservation

$$\mathrm{d}E = T\mathrm{d}S + \mu \mathrm{d}N$$

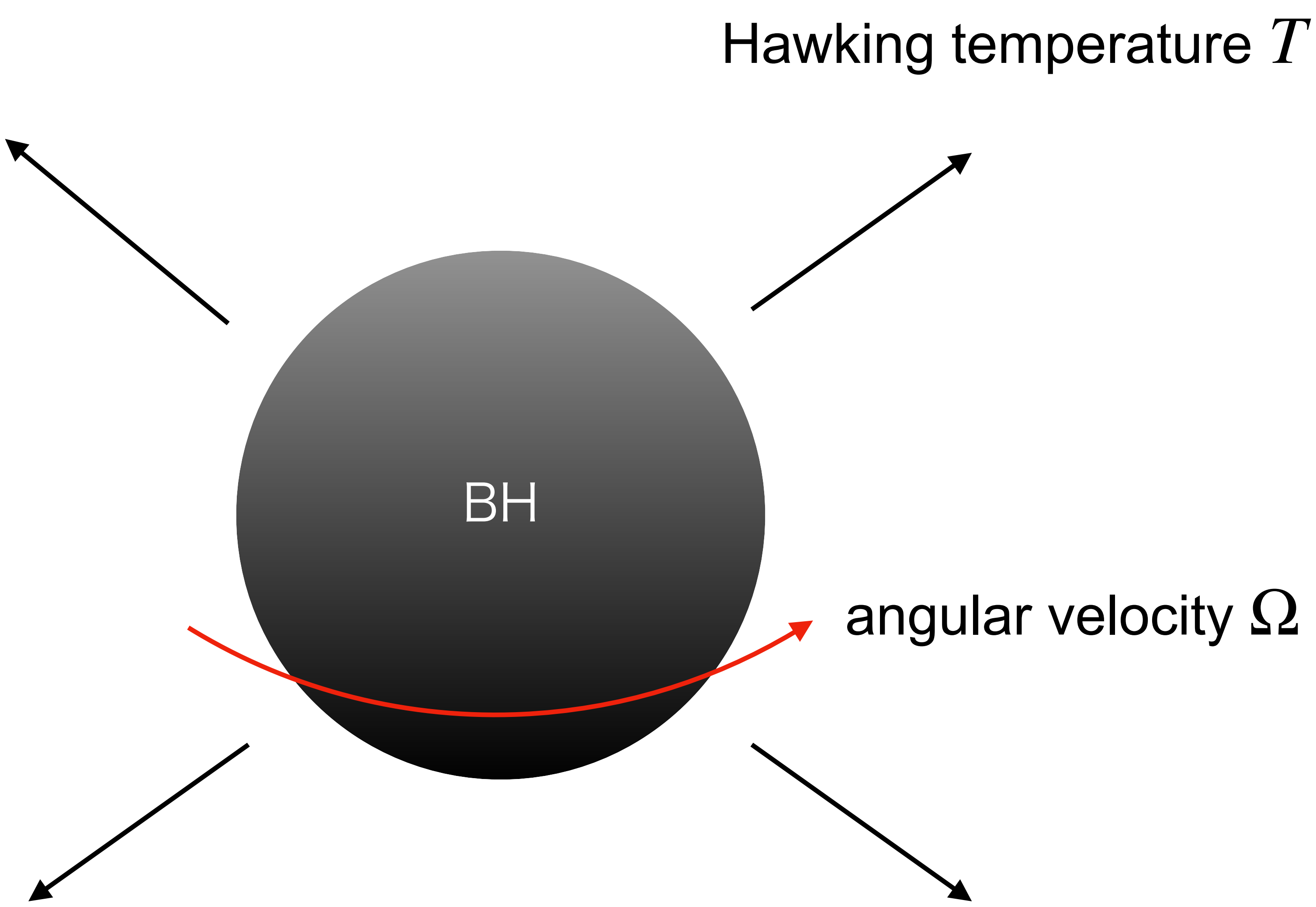
2nd law: Entropy inequality

$$S_t \geq S_0$$

~~**3rd law: $T = 0$ cannot be achieved by finite steps**~~

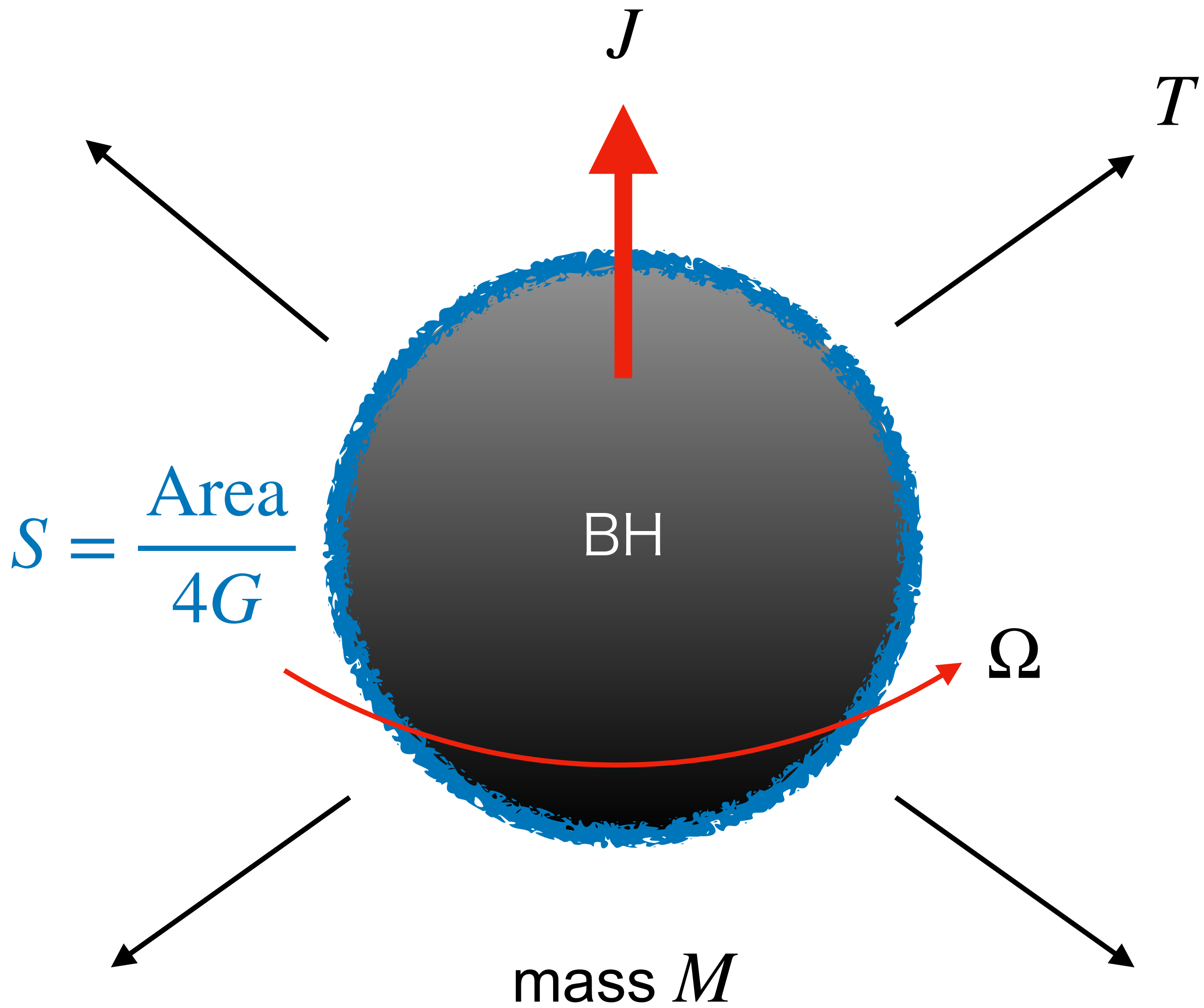
~~Unnecessary for thermodynamics~~

0th law of BHT



Also, electrostatic potential ϕ

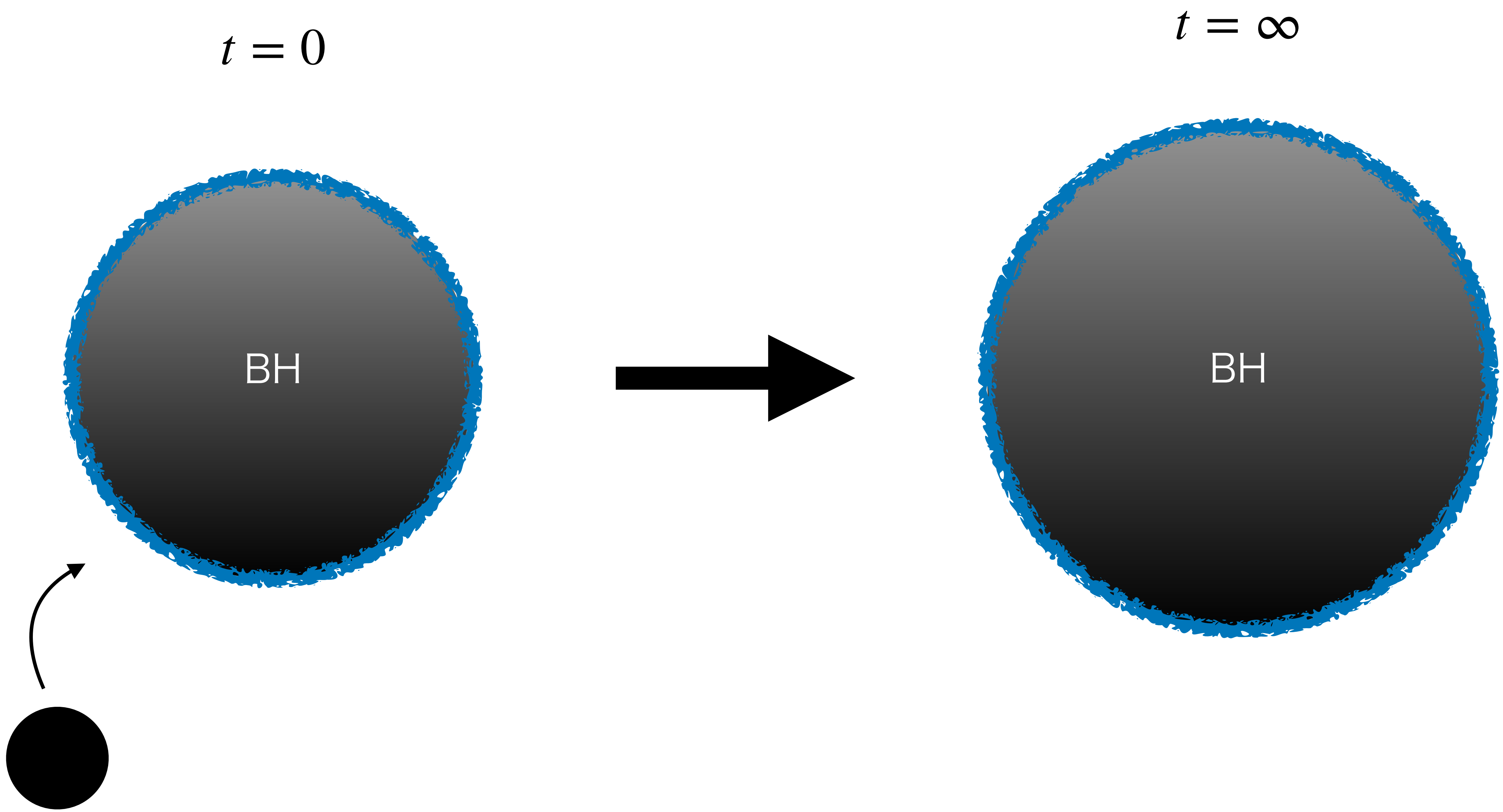
1st law of BHT



$$\mathrm{d}M = T\mathrm{d}S + \Omega \mathrm{d}J + \underline{\phi \mathrm{d}Q}$$

If charged

2nd law of BHT



$$S_0 \leq S_\infty$$

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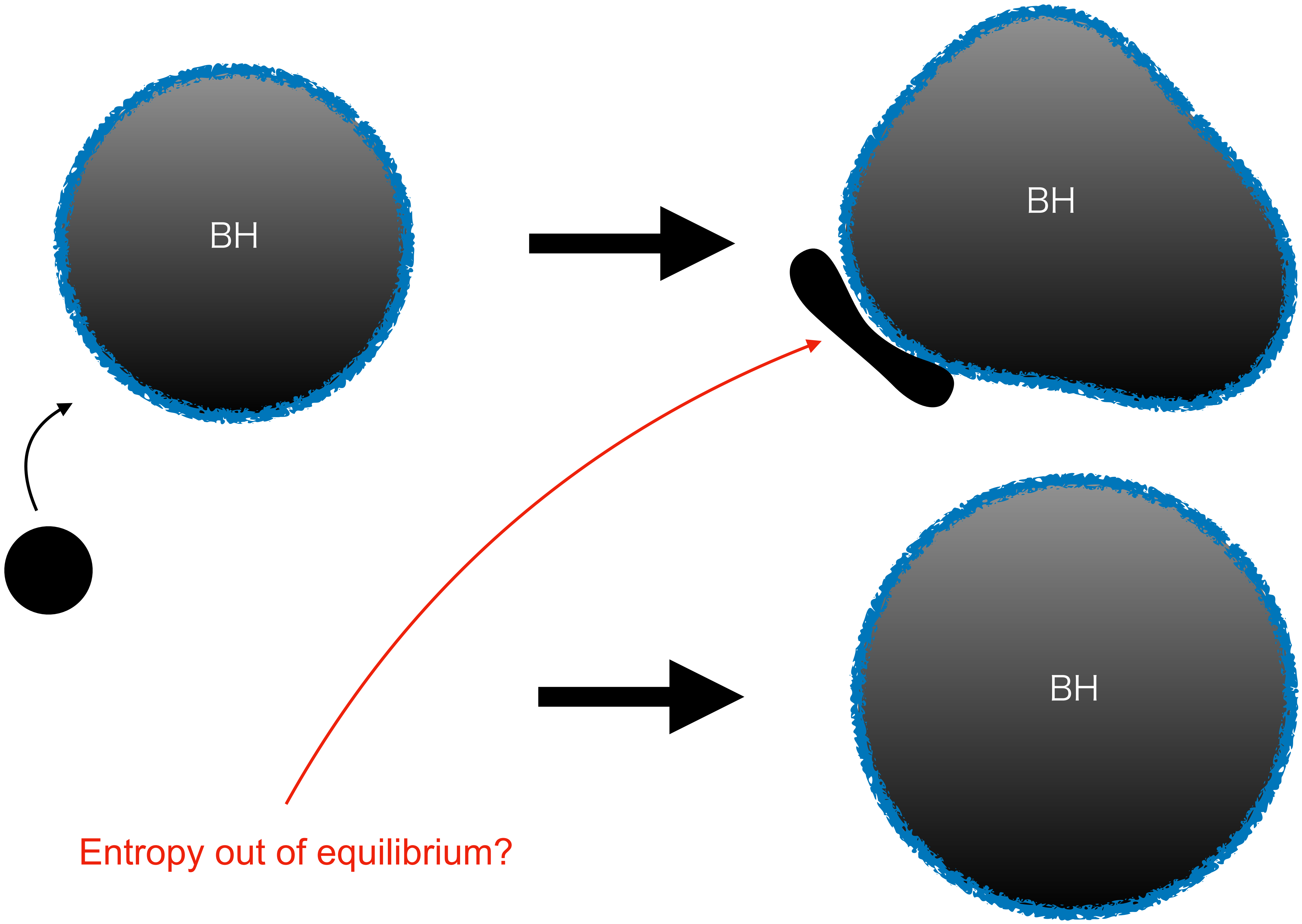
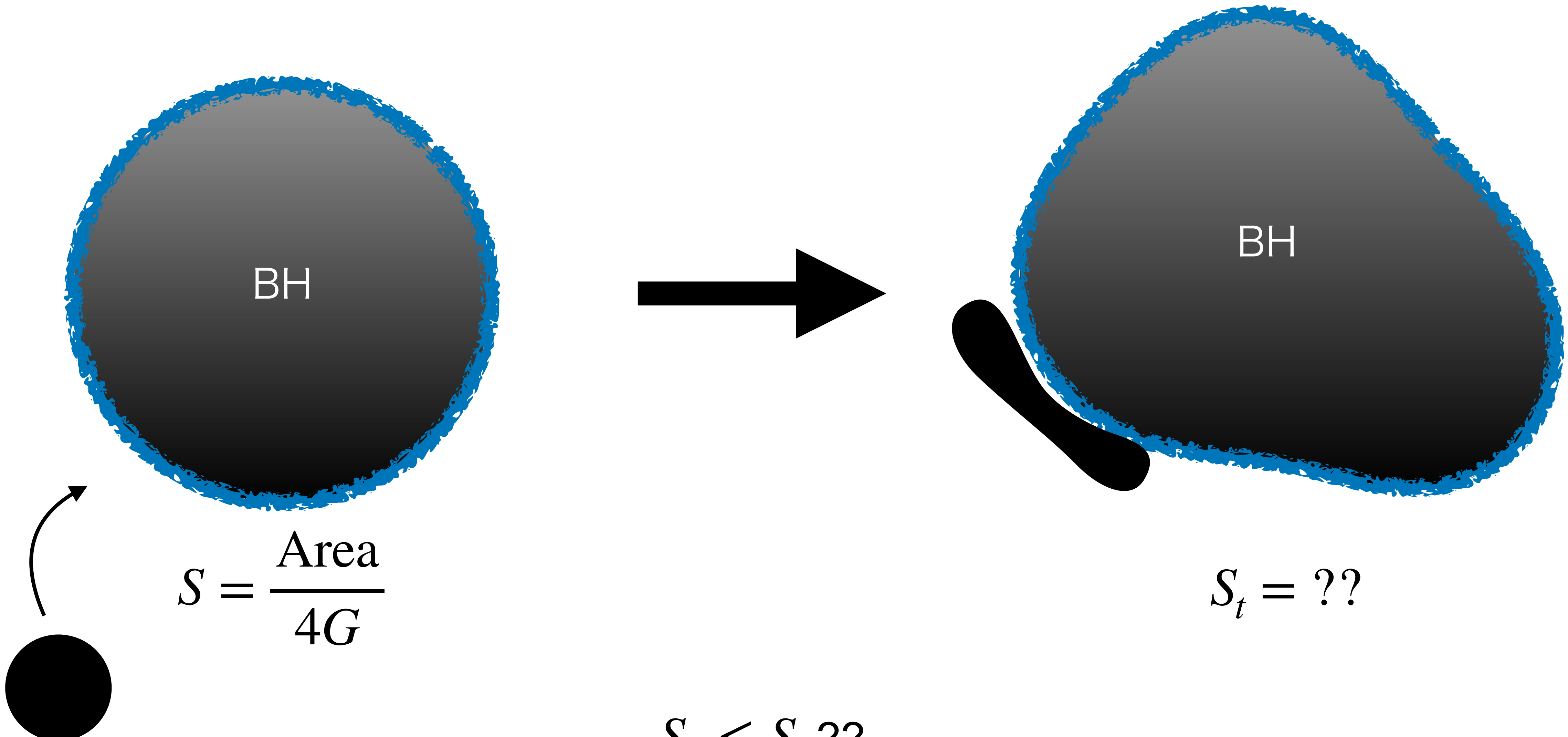
2nd law out of equilibrium



2nd law out of equilibrium

$t = 0$ $\qquad\qquad$ t



$$S = \frac{\text{Area}}{4G} \qquad\qquad S_t = ??$$

$$S_0 \leq S_t \ ??$$

A complete proof not given yet

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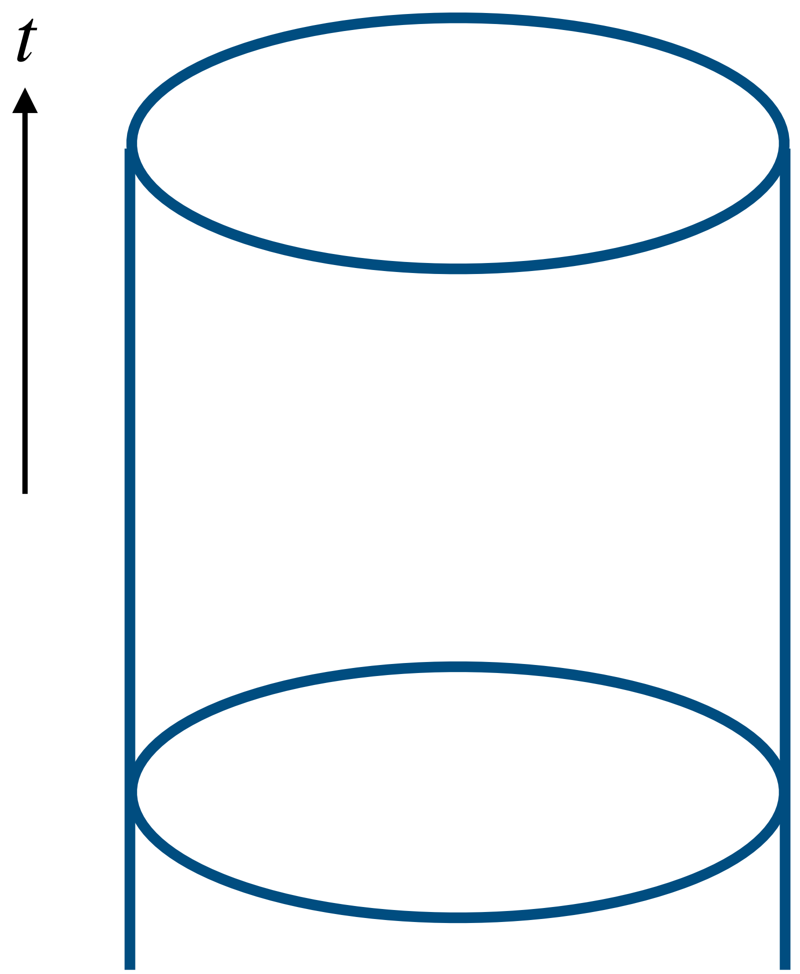
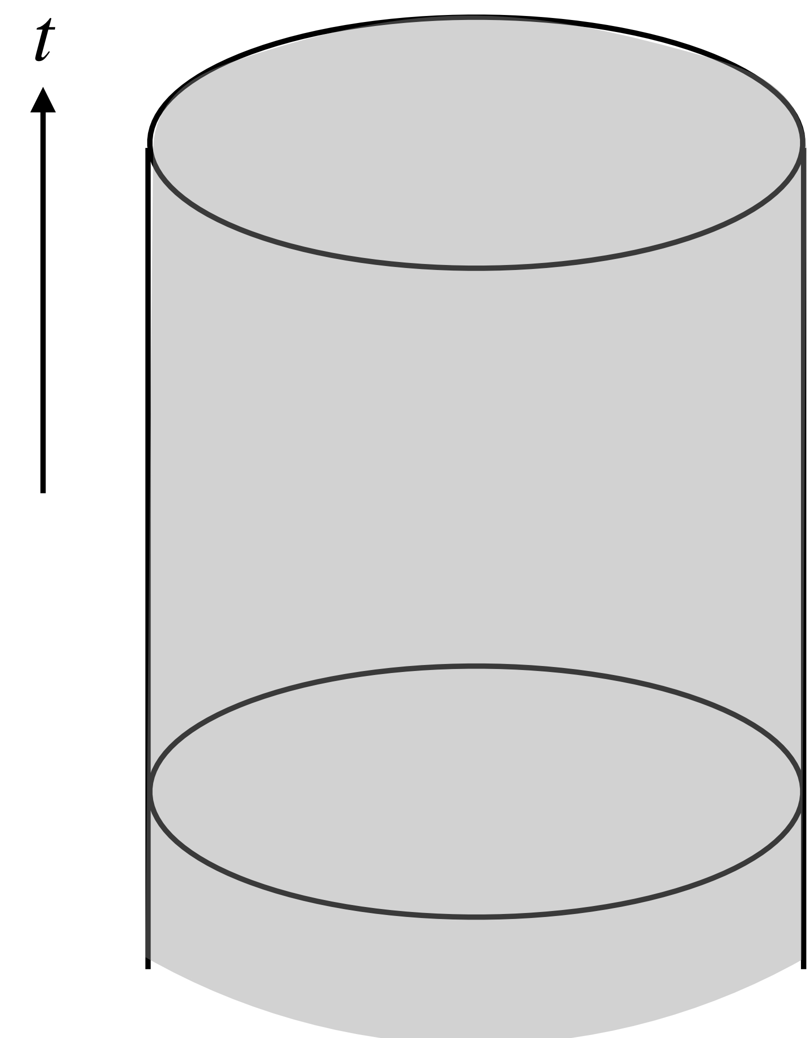
Non-gravity = Gravity

Holography, especially AdS/CFT correspondence



AdS/CFT

=



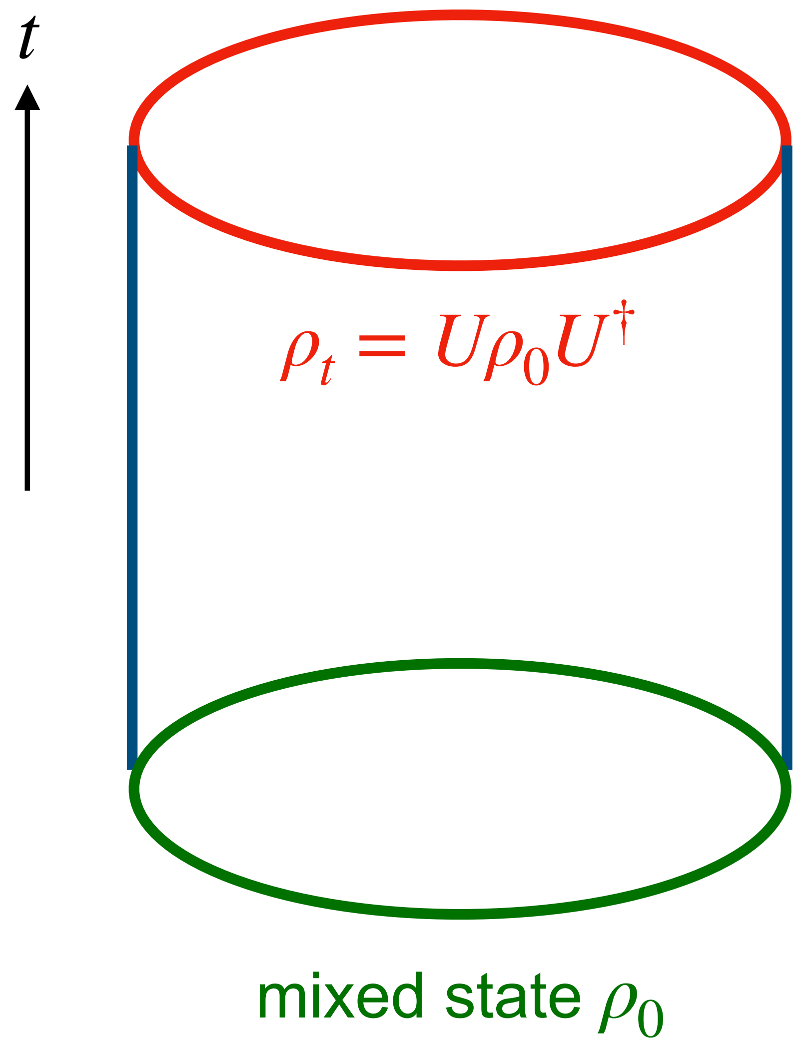
CFT on cylinder

Scale-invariant, QCP

Gravity in the bulk

Negative cosmological constant

Evolving mixed state = Dynamical BH



AdS/CFT
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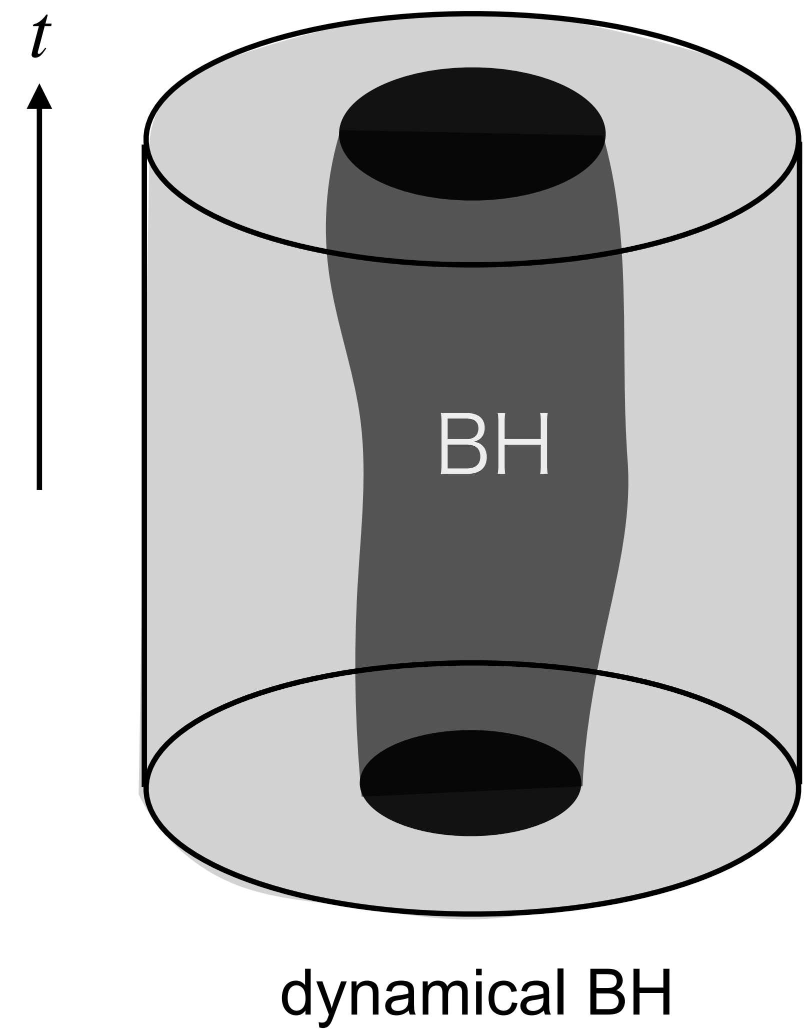

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Thermodynamic entropy in QM

Canonical ensemble

Maximize $S(\rho) = -\operatorname{Tr}\rho\ln\rho$ under $\operatorname{Tr}(H\rho) = E$

$\Rightarrow$ $\rho_{\text{can}} \propto e^{-\beta H}$ ($\beta = \beta(E)$: Lagrange multiplier)

$$S_{\text{can}} = -\operatorname{Tr}\rho_{\text{can}}\ln\rho_{\text{can}}$$

Coarse-grained entropy in general

Coarse-grained state $\rho_{\text{cg},t}$

ρ_t: target state $\{H, O_I(\vec{x})\}$: operator set

$$\rho_{\text{ref},t} := \text{argmax}_\rho \left(-\text{Tr}\rho \ln \rho\right)$$

under $\text{Tr}(\rho H) = \text{Tr}(\rho_t H), \quad \text{Tr}(\rho O_I(\vec{x})) = \text{Tr}(\rho_t O_I(\vec{x}))$

$$\rho_{\text{cg},t} = \frac{1}{Z_t} \exp\left[-\beta_t H + \int \mathrm{d}^{d-1}\vec{x}\, \lambda_t^I(\vec{x}) O_I(\vec{x})\right]$$

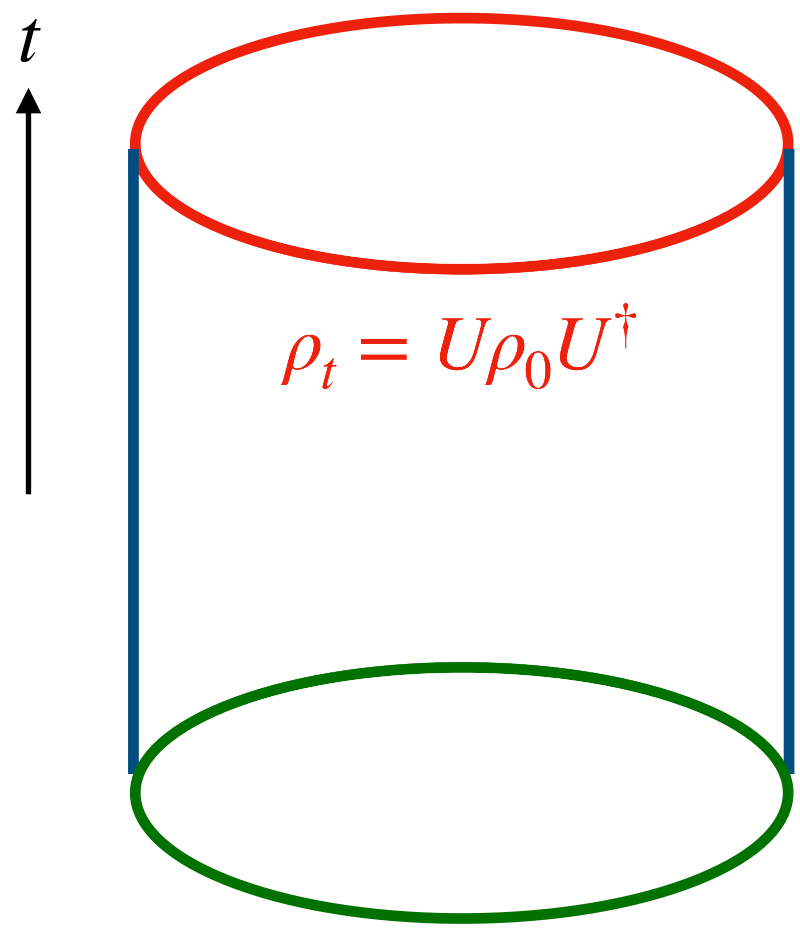
λ_t : Lagrange multipliers

Coarse-grained entropy S_t

$$S_t := -\text{Tr}\rho_{\text{cg},t} \ln \rho_{\text{cg},t} = \beta_t \text{Tr}(\rho_t H) - \int \mathrm{d}^{d-1}\vec{x}\, \lambda_t^I(\vec{x})\, \text{Tr}(\rho_t O_I(\vec{x})) + \ln Z_t$$

2nd law-like inequality

Setup



mixed state ρ_0

$$\rho_t$$

$$\uparrow$$

$$H(t) = H - \int \mathrm{d}^{d-1}\vec{x}\, j^I(t,\vec{x}) O_I(\vec{x})$$

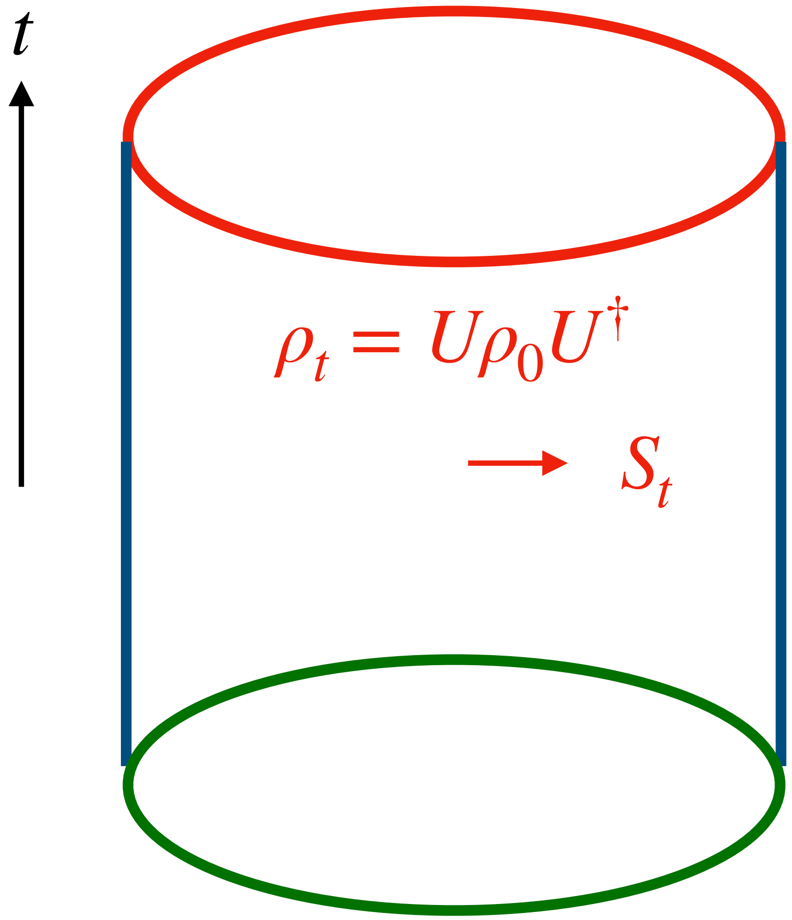
$$\uparrow$$

$$\rho_0 \propto \exp\left[\int \mathrm{d}^{d-1}\vec{x}\, \lambda_0^I(\vec{x}) O_I(\vec{x})\right]$$

Positivity of relative entropy

$$\mathrm{Tr}\, \rho_t \left(\ln \rho_t - \ln \rho_{\mathrm{cg},t}\right) \geq 0 \quad \Longleftrightarrow \quad \text{2nd law} \quad S_t \geq S_0$$

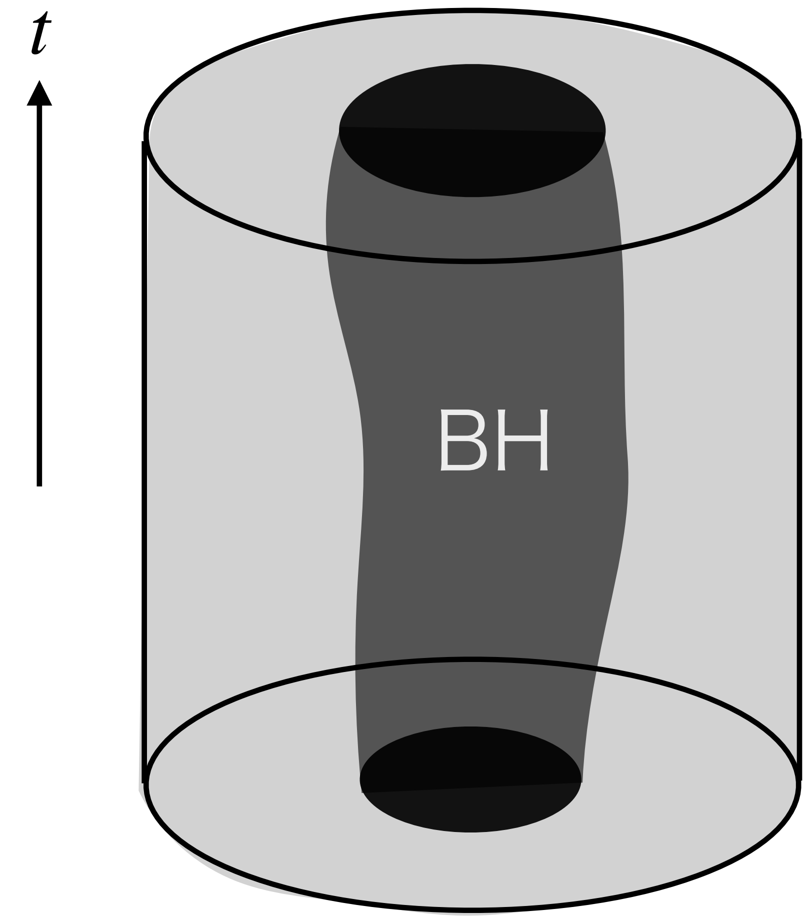
AdS/CFT constrains BH thermodynamics



mixed state ρ_0

$\longrightarrow S_0$

AdS/CFT
=



dynamical BH

$S_0 \leq S_t$ $\xrightarrow{\text{AdS/CFT}}$ Constraint on BH spacetime