Machine-learning emergent spacetime from linear response

Daichi Takeda（RIKEN iTHEMS, Japan）

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Machine Learning for Quantum Fields and Geometry
@UNIST

based on arXiv: 2411.16052 (published)
with K. Hashimoto, K. Matsuo, M. Murata, G. Ogiwara

Self introduction

Daichi Takeda

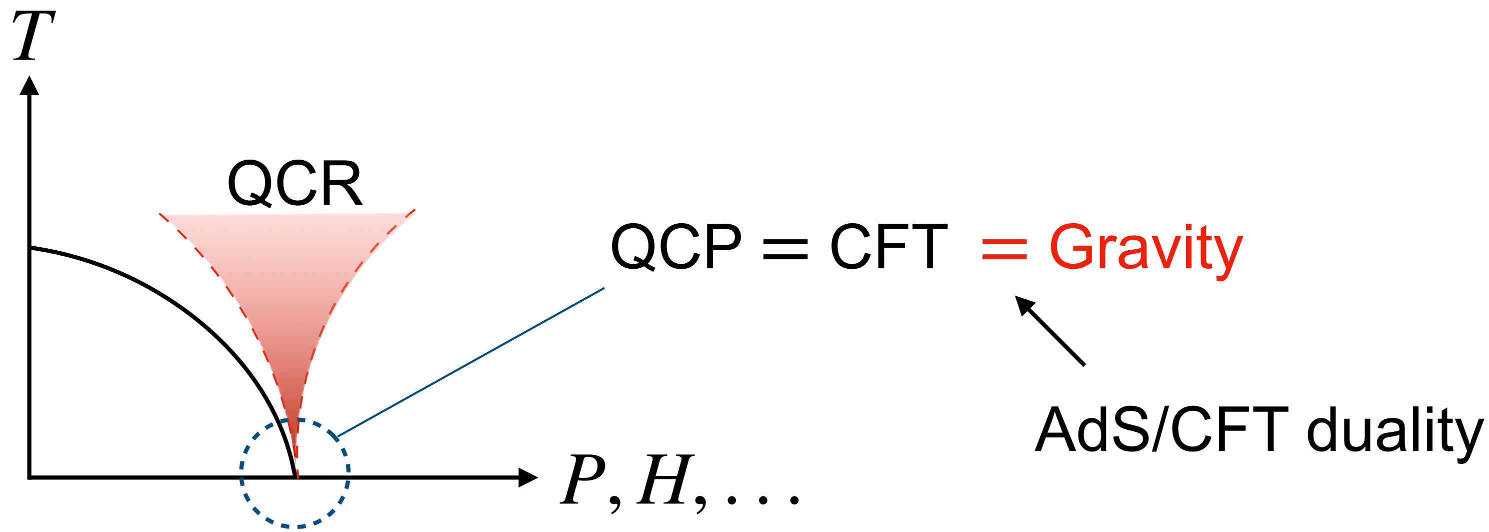
Mar. 2025 Doctor in Kyoto U.

Apr. 2025 RIKEN, iTHEMS (first post doc)

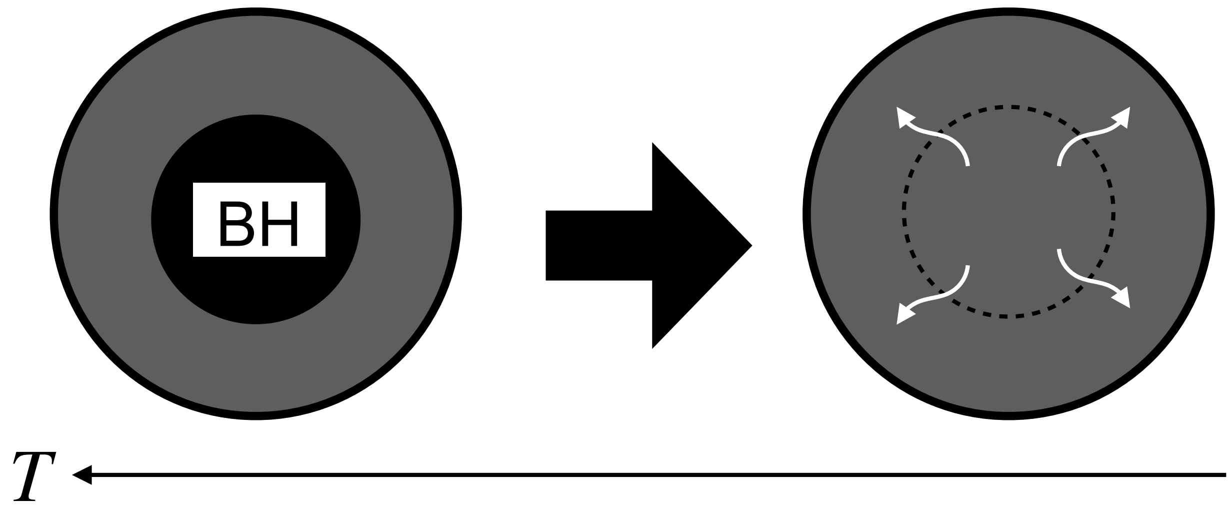
1. H. Hata, **DT**, "Interior Product, Lie Derivative and Wilson Line in the KBc Subsector of Open String Field Theory", JHEP 07 (2021) 117. — SFT
2. **DT**, "Light-cone cuts and hole-ography: explicit reconstruction of bulk metrics", JHEP 04 (2022) 124. — Bulk reconst.
3. H. Hata, **DT**, J. Yoshinaka, "Generating string field theory solutions with matter operators from KBc algebra",
   PTEP 2022 9, 093B09 (2022) — SFT
4. K. Hashimoto, **DT,** K. Tanaka, S. Yonezawa**,** "Spacetime-emergent ring toward tabletop quantum gravity experiments",
   Phys. Rev. Res. 5 (2023) 2, 023168. — AdS/CMT
5. K. Sugiura, **DT**, "Bulk reconstruction of AdSd+1 metrics and developing kinematic space", JHEP 06 (2023) 035. — Bulk reconst.
6. S. Kinoshita, K. Murata, **DT**, "Shooting null geodesics into holographic spacetimes", JHEP 10 (2023) 074. — AdS/CMT
7. M. Bamba, K. Hashimoto, K. Murata, **D. Takeda**, D. Yamamoto,
   "Spacetime-localized response in quantum critical spin systems: Insights from holography",
   Phys. Rev. D 109, 126003. — AdS/CMT
8. **DT**, "Coarse-graining black holes out of equilibrium with boundary observables on time slice", JHEP 05 (2024) 319 — BH thermo
9. K. Hashimoto, K. Matsuo, M. Murata, G. Ogiwara and **DT**, "Machine-learning emergent spacetime from linear response in future tabletop quantum gravity experiments", Mach.Learn.Sci.Tech. 6 (2025) 1, 015030 — Today's topic
10. T. Shigemura, K. Shimizu, S. Sugishita, **DT**, T. Yoda, "Heat and work in black hole thermodynamic via holography", JHEP 05 (2025) 069 — BH thermo
11. Takanori Ishii, **Daichi Takeda**, "Lindblad dynamics in holography", Phys.Rev.D 112 (2025) 4, 046020. — Open AdS/CFT

Quantum gravity experiments in laboratories

Gravitational theories in laboratories?

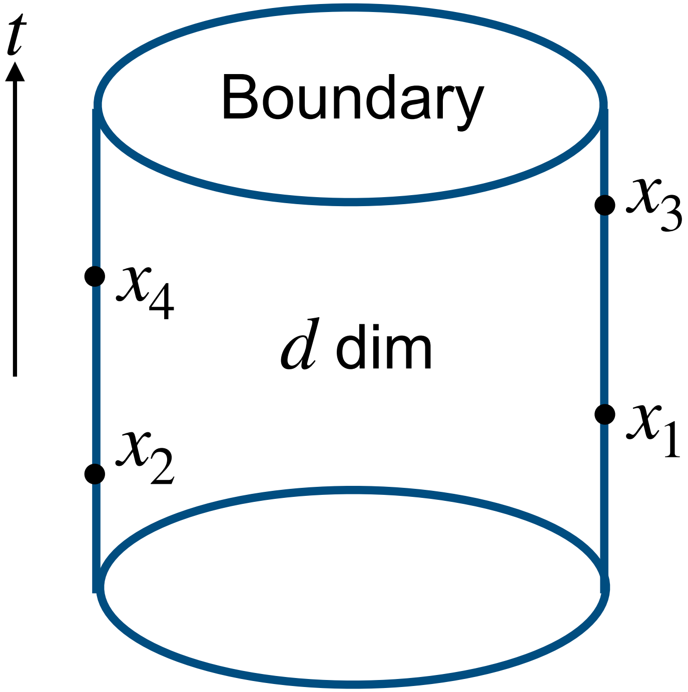


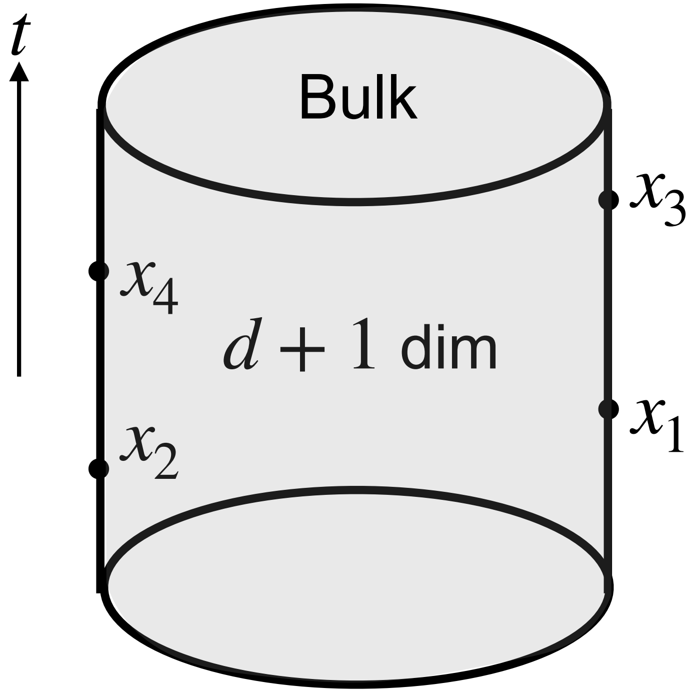
Quantum gravity appears when a BH evaporates



AdS theory dual to a given material?

Find an AdS theory phenomenologically





$$\langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle \overset{\text{GKPW}}{=} \langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle$$

for equilibrium states — on fixed backgrounds

What is the metric corresponding to a given CFT state?

Boundary linear response → bulk metric

Linear response is equivalent to the retarded Green's function

$$H(t) = H + V(t), \quad V(t) := \int \mathrm{d}^{d-1}\vec{x}\ \underline{J(t,\vec{x})}O_S(\vec{x})$$

support: $t > 0$

$\boxed{\text{Interaction picture}}$

$$\rho_I(t) = \mathrm{T}\exp\left[-i\int_0^t \mathrm{d}s\ \mathrm{ad}[V_I(s)]\right]\rho_0$$ Gibbs state

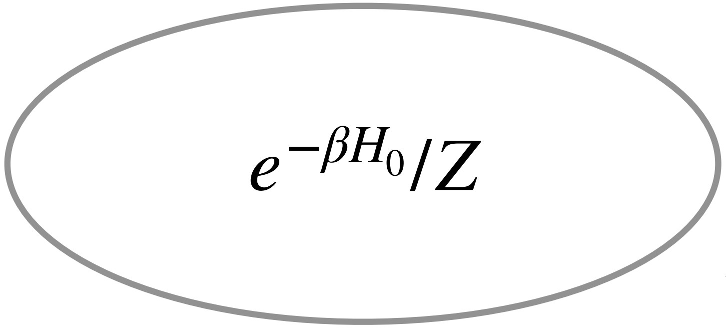
$$\langle O_I(x)\rangle_J \simeq \cancel{\mathrm{Tr}[\rho_0 O_I(x)]}$$ 0 (assumption)

$$+\int \mathrm{d}^d y \left\{\underbrace{-i\Theta(x^0-y^0)\mathrm{Tr}\left(\rho_0[O_I(x),O_I(y)]\right)}_{=:\, G_R(x,y)}\right\}J(y)$$

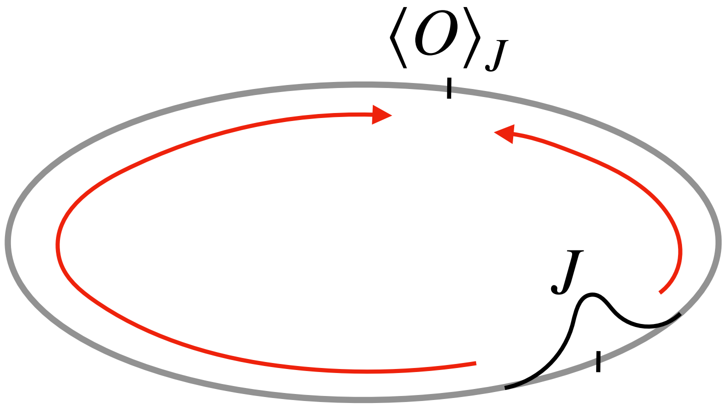
Linear response

Boundary linear response → bulk metric

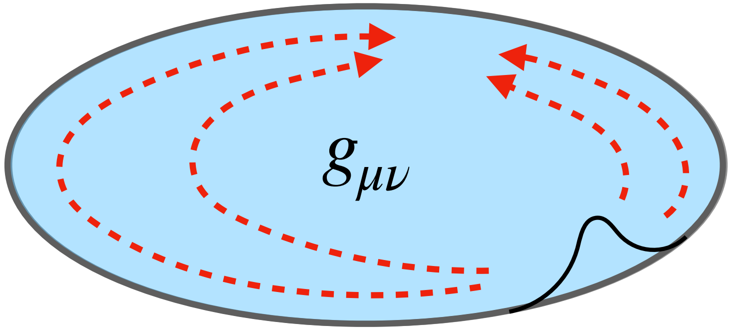
Prepare ring-shaped material
in Gibbs state



Get linear response data

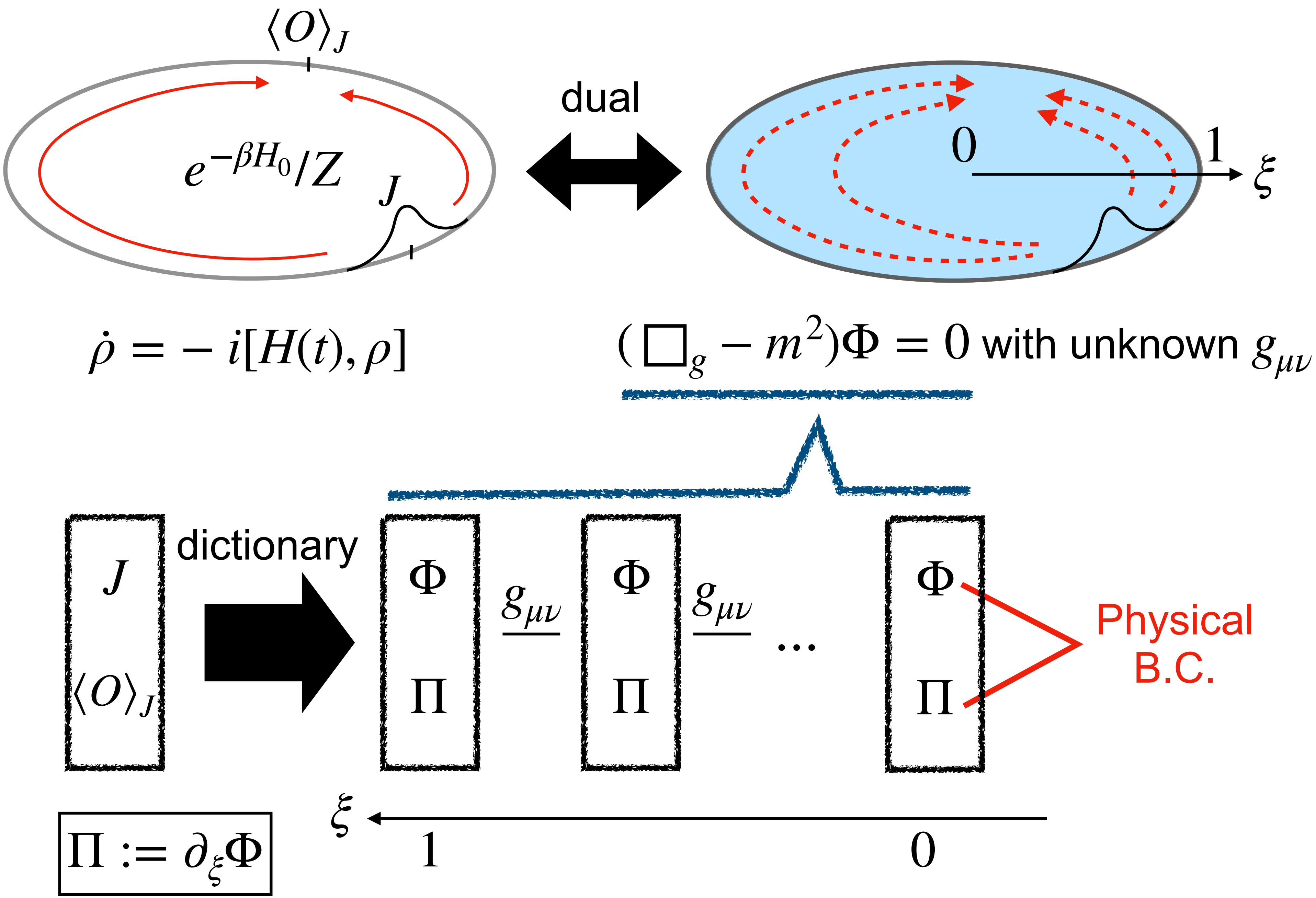


Find the bulk metric that reproduces the data



Machine-learning

Machine-learning $g_{\mu\nu}$ from linear response



Machine-learning emergent spacetime from linear response

1. The forward problem:
   Boundary linear response from bulk

2. The inverse problem:
   Bulk metric from boundary linear response

3. NN solves the inverse problem

4. Demonstration:
   NN reproduces BTZ metric

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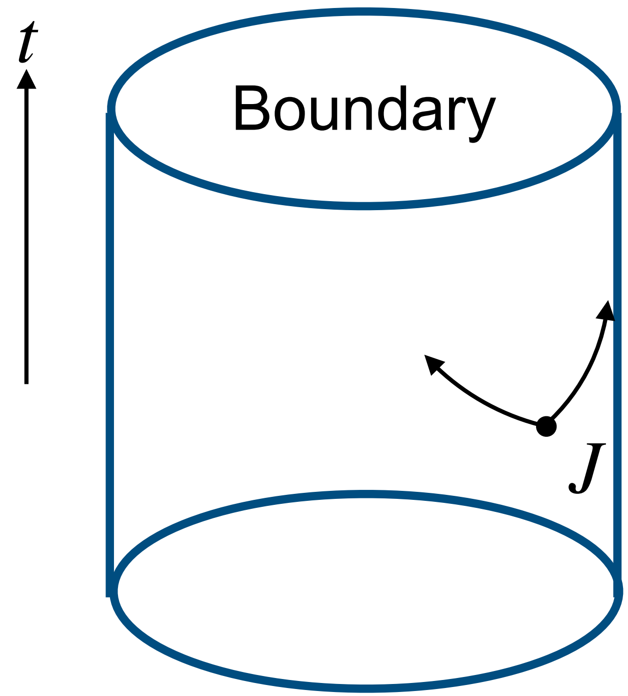
AdS/CFT: Generating functionals are equal

AdS/CFT correspondence (large N)

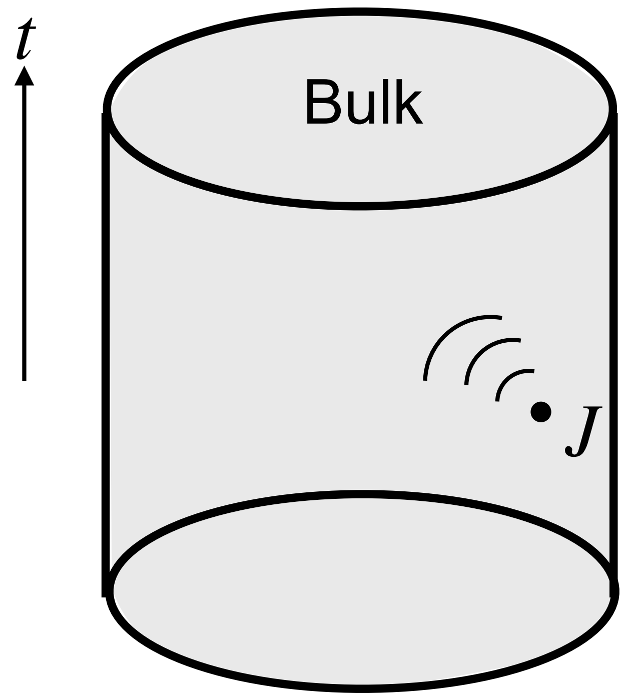
$$\int \mathscr{D}\phi\, e^{iI_{\mathrm{CFT}}[\phi]+i\int \mathrm{d}^d x\, J(x) O_\Delta(x)} = e^{iI_{\mathrm{AdS}}[\Phi_{\mathrm{cl}}]}\Big|_{\Phi\sim r^{\Delta-d}J}$$

CFT (d dim)



Quantum theory

AdS gravity ($d+1$ dim)



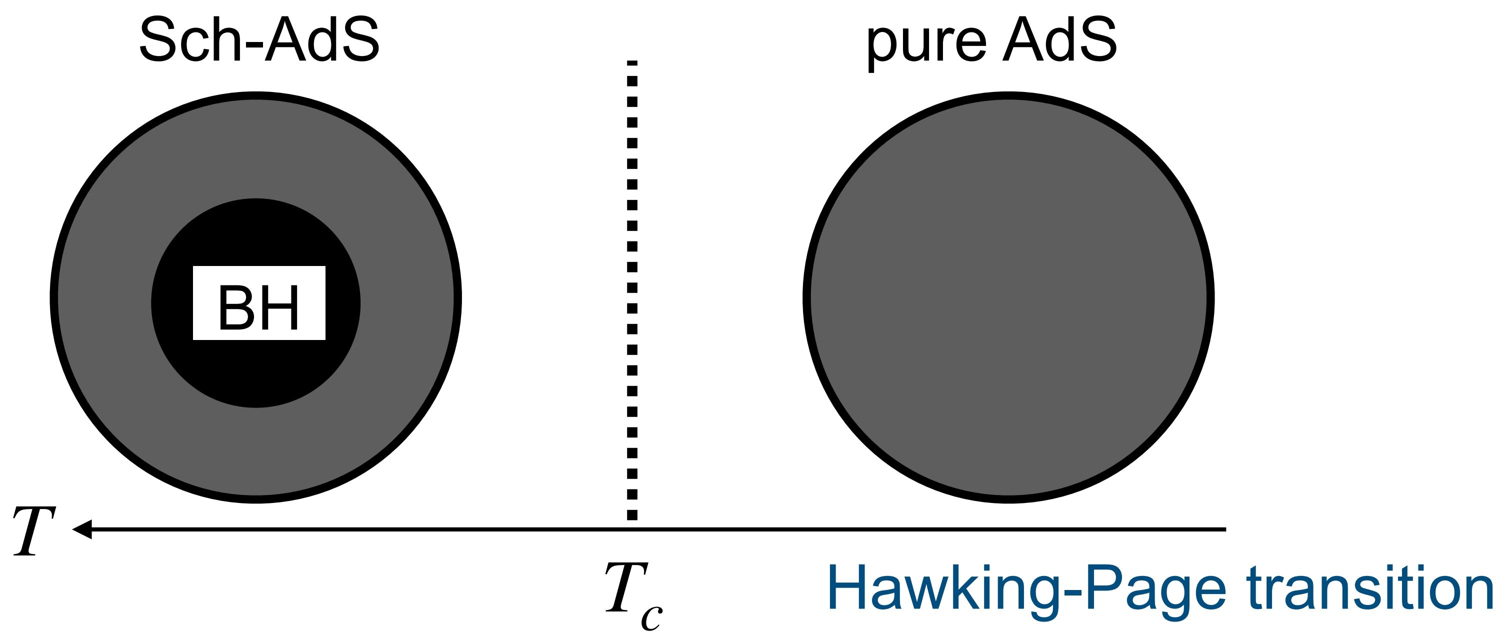
Classical field theory

The bulk configuration dual to Gibbs state

Apply the dictionary in Euclidean signature

$$\mathrm{Tr} e^{-\beta H} = \int \mathscr{D}\phi \, e^{-I_{\mathrm{CFT}}[\phi]} = e^{-I_{\mathrm{AdS}}[\Phi_{\mathrm{cl}}]} \Big|_{\Phi \sim r^{\Delta - d} \times 0}$$

If I_{AdS} is Einstein, we find two solutions:

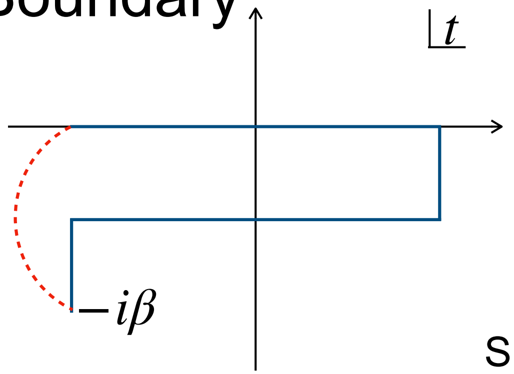


In general theories, we do not know if the bulk has a BH.
But, $g_{\mu\nu}$ is static and spherically symmetric.

Linear response around Gibbs state

Apply the dictionary for Schwinger-Keldysh contour

Boundary



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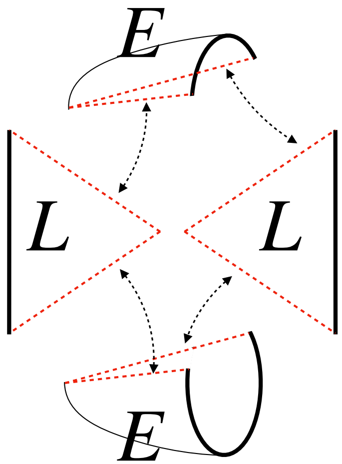
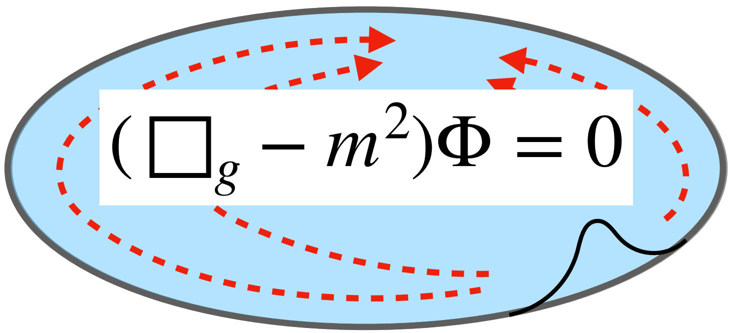
Bulk

(BH case)



Skenderis, van Rees (2008)

For linear response, this reduces to …



$$(\Box_g - m^2)\Phi = 0$$

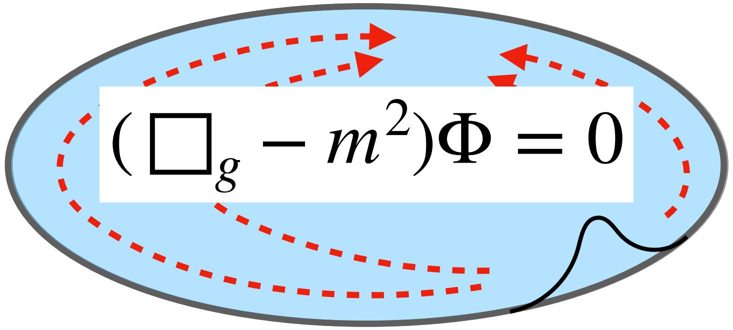
$$\Phi \underset{r\to\infty}{\sim} r^{\Delta-d} J$$

When interested in $\mathscr{O}(J^1)$,

we need free Φ on fixed $g_{\mu\nu}$

The one dual to the Gibbs state

Linear response around Gibbs state



$$\Phi \underset{r\to\infty}{\sim} r^{\Delta-d} J$$

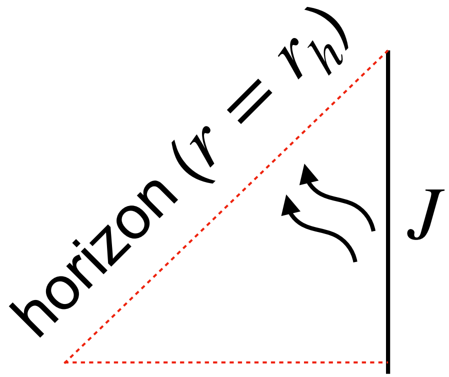
One more B.C. is necessary

$\Phi = 0$ for $t \leq 0$ … $*$

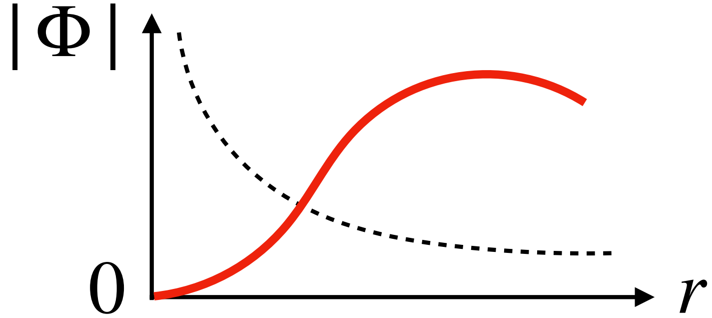
(retarded B.C.)

+ retarded $i\epsilon$-presc.

If BH exists, $*$= ingoing B.C.



If not, $*$ = regular B.C.

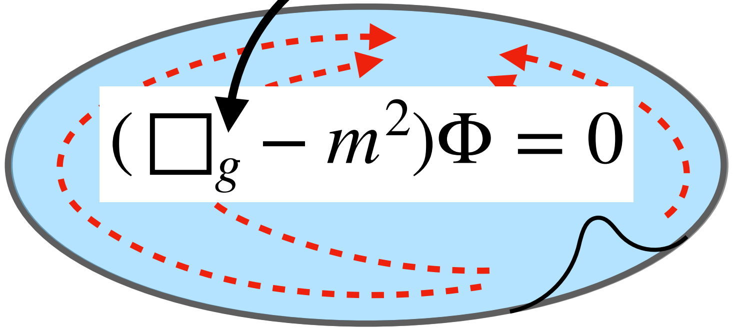


Then, how to extract $\langle O \rangle_J$?

$$\Phi \underset{r\to\infty}{\sim} r^{\Delta-d} J + r^{-\Delta} \underline{\langle O \rangle_J}$$

Forward problem: Boundary linear response from bulk

$$\mathrm{Tr}e^{-\beta H} = \int \mathscr{D}\phi \, e^{-I_{\mathrm{CFT}}[\phi]} = e^{-I_{\mathrm{AdS}}[\Phi_{\mathrm{cl}}]}\Big|_{\Phi \sim r^{\Delta-d}\times 0} \quad \Rightarrow \quad g_{\mu\nu}$$



$$(\Box_g - m^2)\Phi = 0 \quad \Rightarrow \quad \Phi$$

with $\Phi \underset{r\to\infty}{\sim} r^{\Delta-d}J$ and retarded B.C.

$$\Phi \underset{r\to\infty}{\sim} r^{\Delta-d}J + r^{-\Delta}\underline{\langle O\rangle_J}$$

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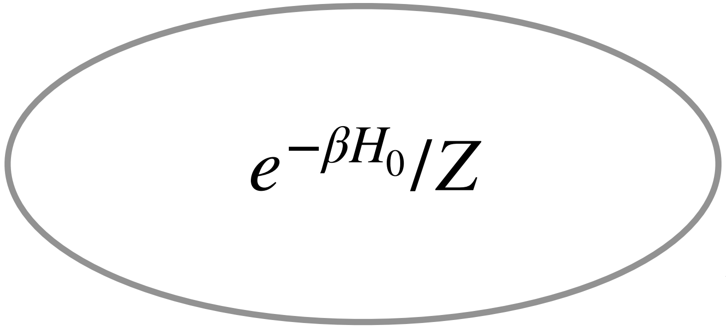
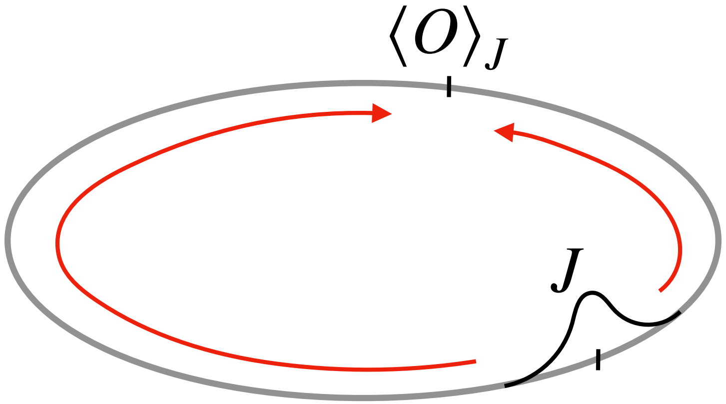
4. Demonstration:
   NN reproduces BTZ metric

Boundary linear response → bulk metric

Prepare ring-shaped (1+1d)
material in Gibbs state

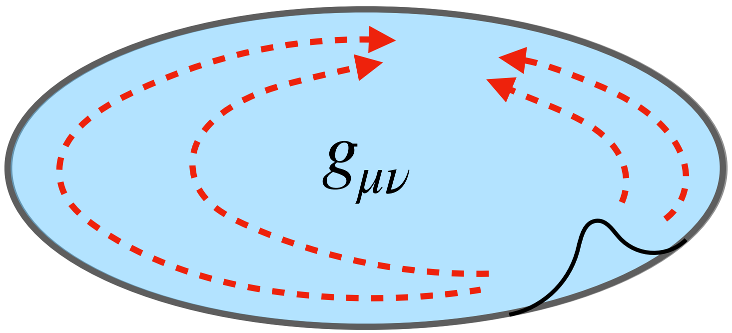


Get linear response data
(by experiment or simulation)



O : scalar primary

Find the bulk metric that reproduces the data



$g_{\mu\nu}$ has two components

Static and spherically symmetric metric

$$ds^2 = g_{tt}(r)\mathrm{d}t^2 + g_{rr}(r)\mathrm{d}r^2 + g_{\theta\theta}(r)\mathrm{d}\theta^2$$

Residual gauge: $r = r(\xi)$

Gauge fix: $g^{\xi\xi}\sqrt{-g} \propto \xi^{-1}$ $\qquad (\xi \in [0,1])$

Two independent components

$$\Xi(\xi) := g_{\xi\xi}g^{tt}, \qquad \Theta(\xi) := g_{\xi\xi}g^{\theta\theta}$$

(Einstein is not assumed)

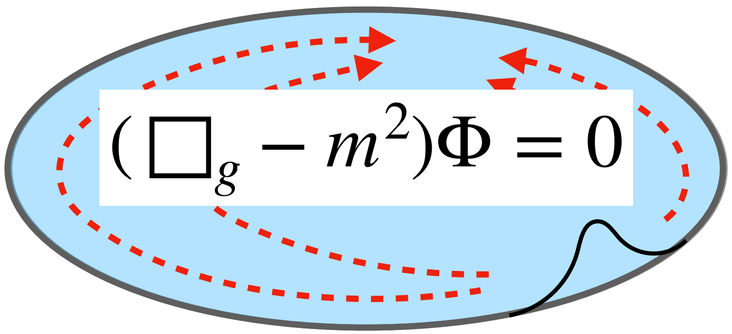
Bulk theory: a probe free scalar field

We have the data of $\langle O \rangle_J$ under

$$H(t) = H + V(t), \quad V(t) := \int \mathrm{d}^{d-1}\vec{x}\; J(t,\vec{x}) O_S(\vec{x})$$

O: scalar primary with scale-dim Δ

J: small external source



Φ: probe free scalar

with mass

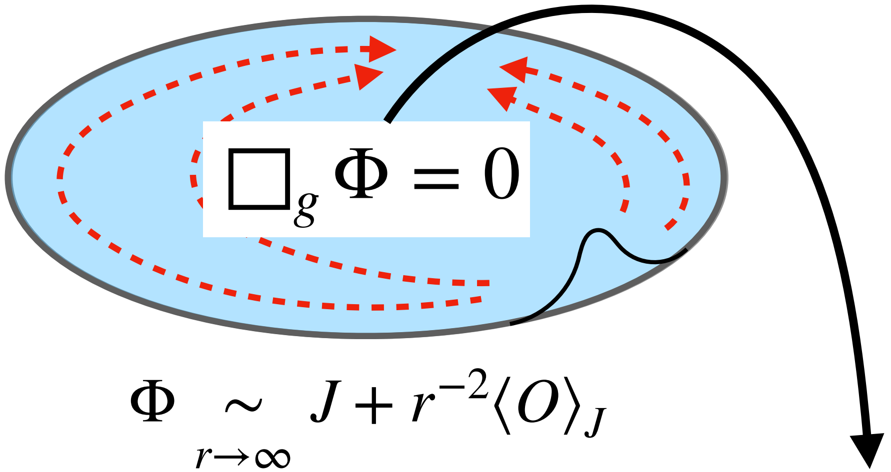
$$m^2 = \Delta(\Delta - 2)$$

$$\Phi \underset{r\to\infty}{\sim} r^{\Delta-2} J + r^{-\Delta} \langle O \rangle_J \longleftarrow$$

Now, J and $\langle O \rangle_J$ are given

Klein-Gordon eq with (Ξ, Θ)

Below, $\Delta = 2\ (m^2 = 0)$ for simplicity



$$\Box_g \Phi = 0$$

$$\Phi \underset{r\to\infty}{\sim} J + r^{-2}\langle O\rangle_J$$

In Fourier space
$$\Phi = e^{-i\omega t + ik\theta}\Phi(\xi)$$

$$Z'(\xi) = \begin{pmatrix} 0 & 1 \\ \omega^2\Xi(\xi)+k^2\Theta(\xi) & -1/\xi \end{pmatrix} Z(\xi)$$

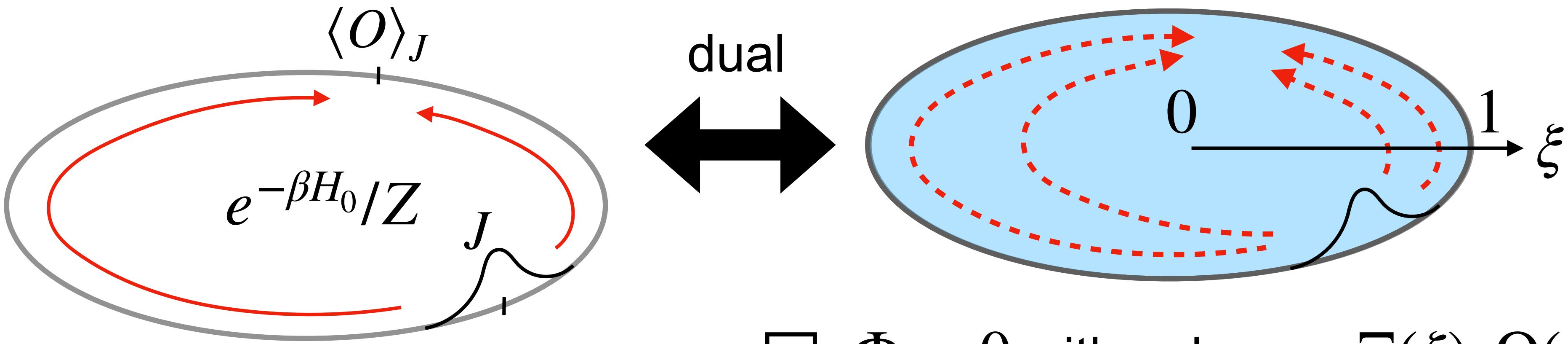
$$Z(\xi) = \begin{pmatrix} \Phi(\xi) \\ \Phi'(\xi) \end{pmatrix}$$

$$\Xi(\xi) := g_{\xi\xi}g^{tt}, \qquad \Theta(\xi) := g_{\xi\xi}g^{\theta\theta}$$

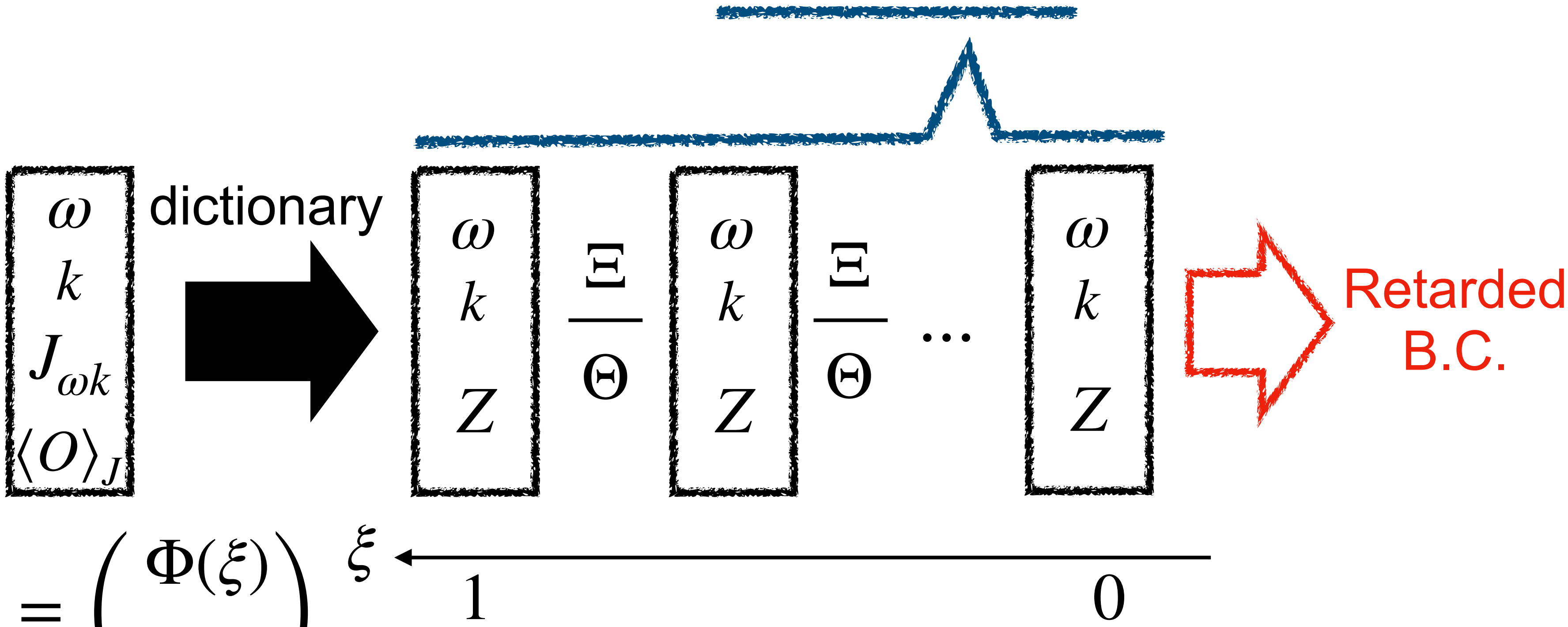
Unknown functions

$Z(\xi = 0)$ must satisfy the retarded B.C.

Inverse problem:
Bulk metric from boundary linear response



$\Box_g \Phi = 0$ with unknown $\Xi(\xi), \Theta(\xi)$



$$Z(\xi) = \begin{pmatrix} \Phi(\xi) \\ \Phi'(\xi) \end{pmatrix}$$

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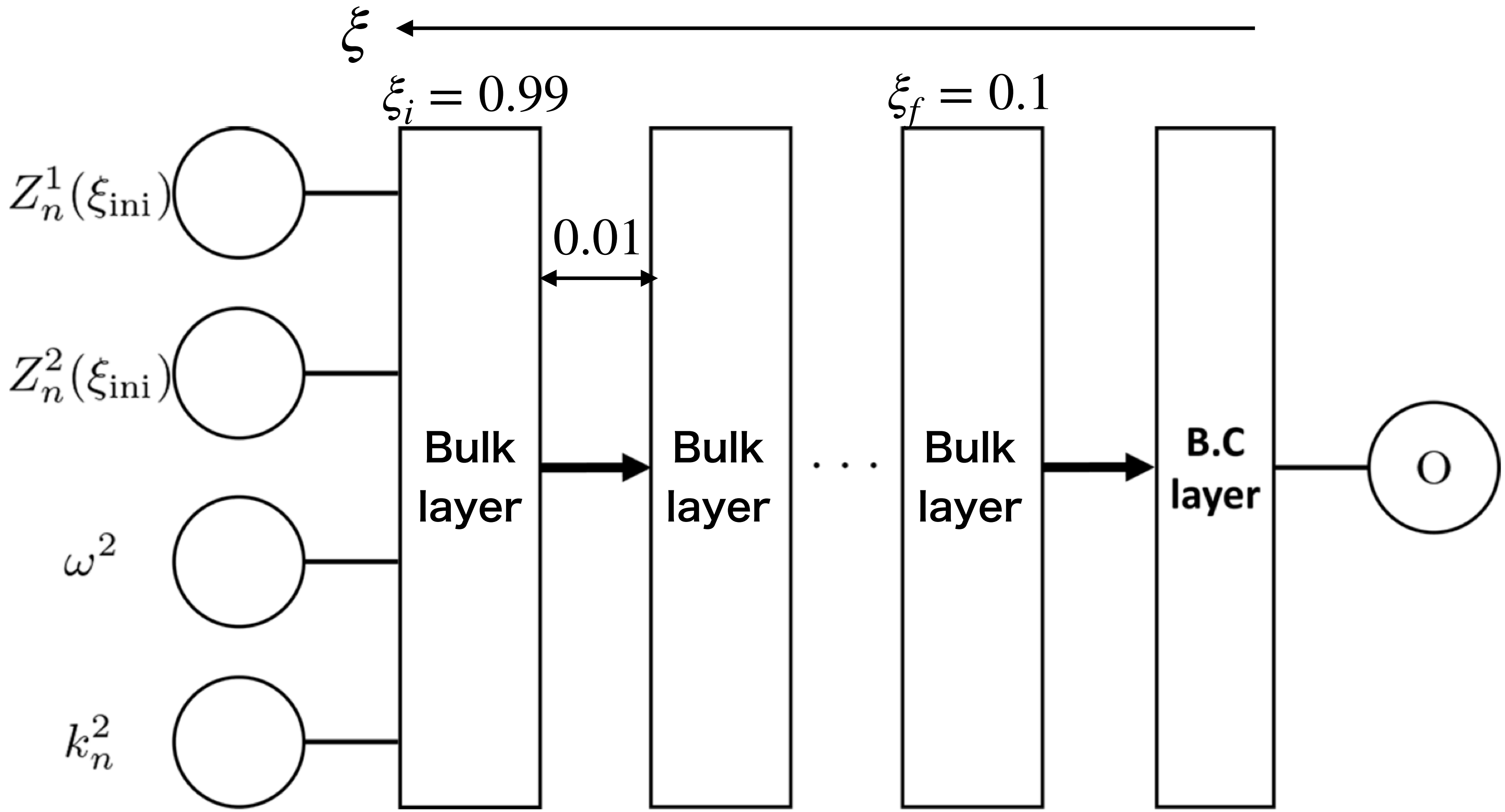
4. Demonstration:
   NN reproduces BTZ metric

Solve with binary classification

Simple version



Data of the linear response (true and false)

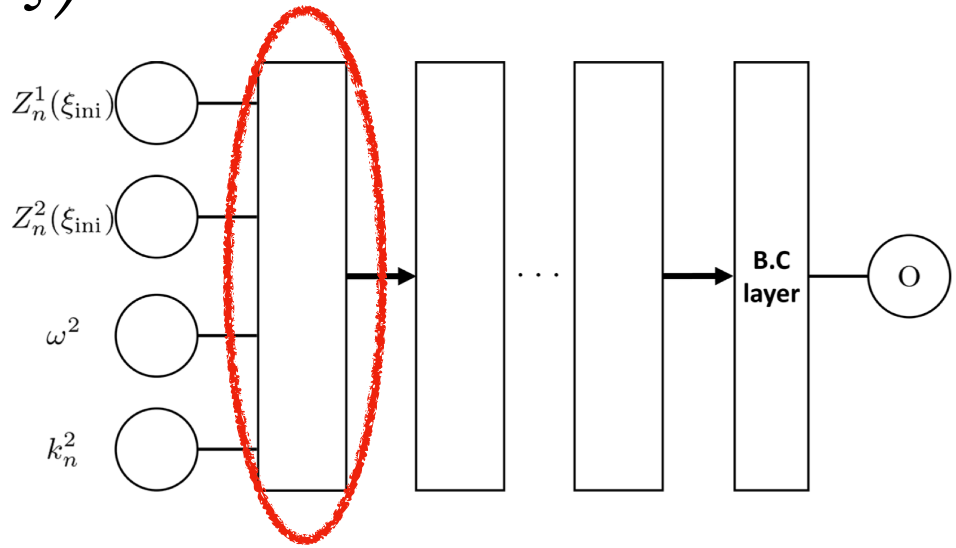
Discretized EOM with weight (Ξ, Θ)

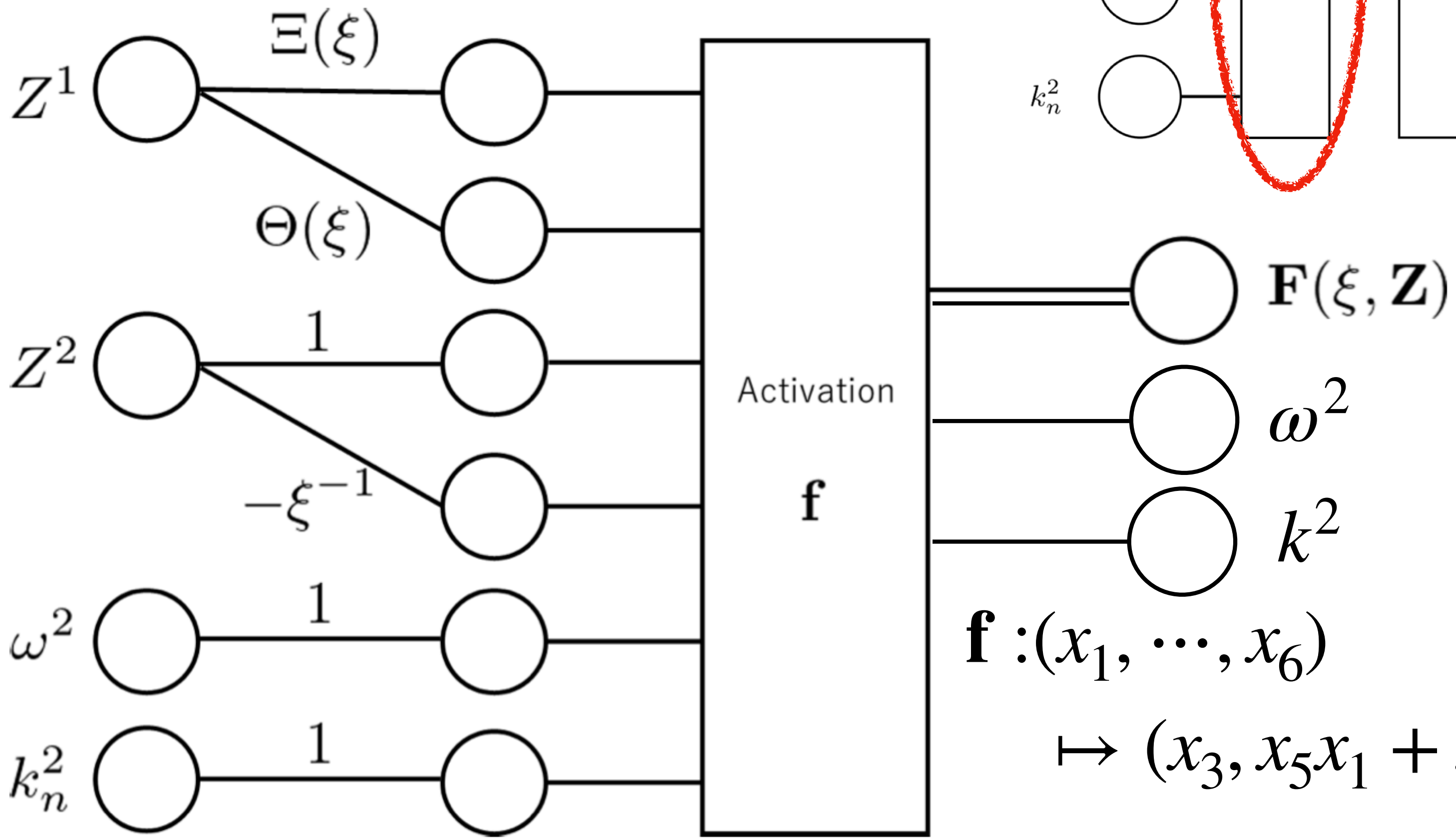
Ask if output matches B.C. with weight ρ

Bulk layer

$$Z'(\xi) = \begin{pmatrix} 0 & 1 \\ \omega^2 \Xi(\xi) + k^2 \Theta(\xi) & -1/\xi \end{pmatrix} Z(\xi) =: F(\xi, Z(\xi))$$

$$Z(\xi) = \begin{pmatrix} \Phi(\xi) \\ \Phi'(\xi) \end{pmatrix}$$



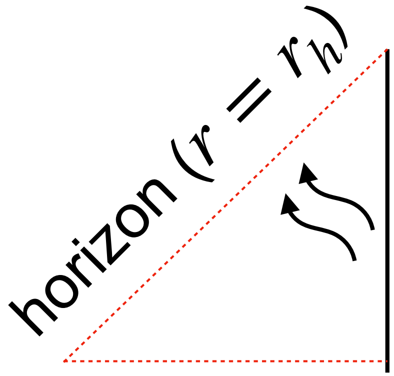


$$\mathbf{f} : (x_1, \cdots, x_6) \mapsto (x_3, x_5 x_1 + x_6 x_2 + x_4, x_5, x_6)$$

Boundary condition layer

Typical behavior at $\xi \sim 0$

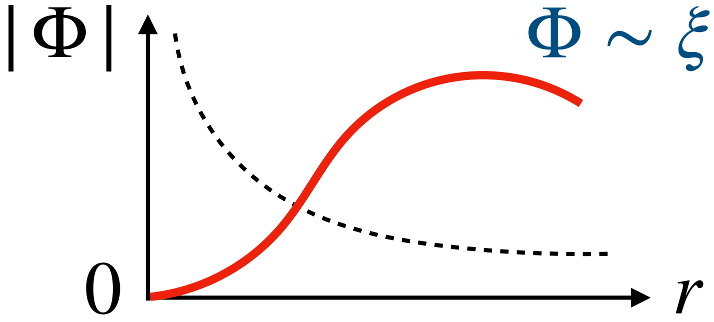
If BH exists, $*$= ingoing B.C.

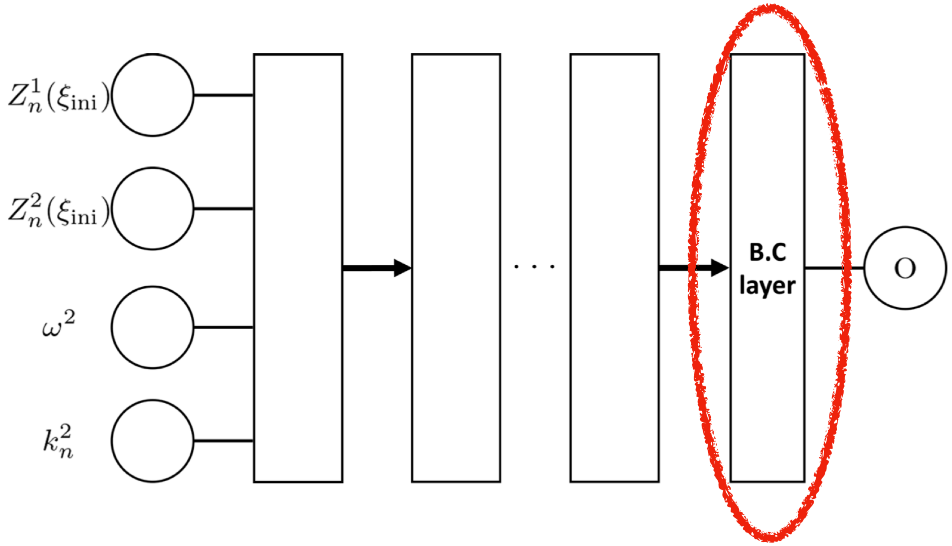


$\Phi \sim \xi^{-ia\omega}$

If not, $*$ = regular B.C.

$\Phi \sim \xi^{b|k|}$





$$O = \left| \frac{\xi_f Z^1(\xi_f) + \rho Z^2(\xi_f)}{\sqrt{\xi_f Z^1(\xi_f) + \rho Z^2(\xi_f) + \epsilon}} \right|$$

$\rho = ia\omega + b|k|$

To be optimized

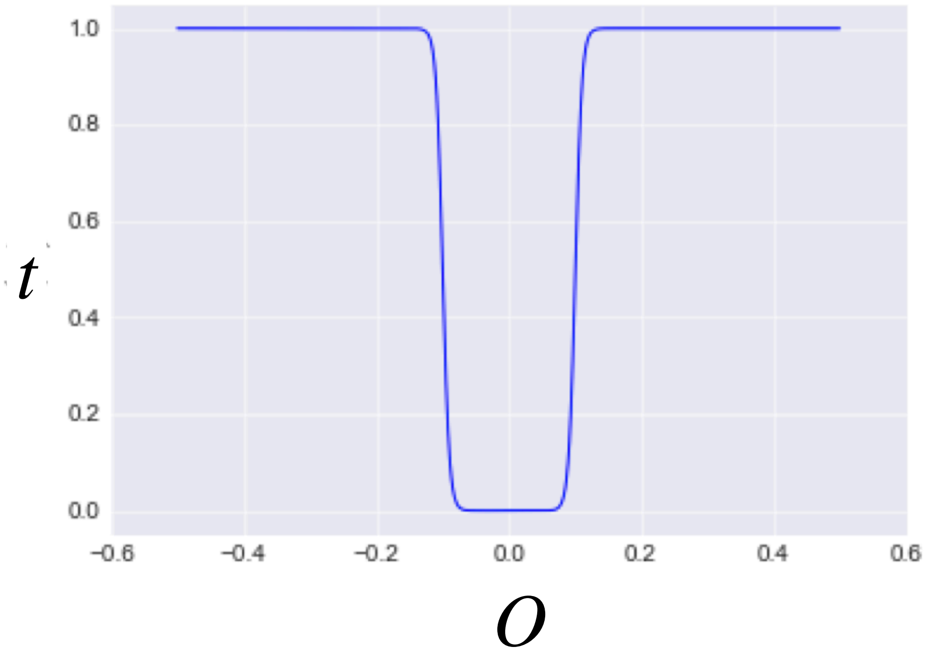
O is expected to be small for true data

Loss function

$$\text{Loss} = -\,t_{\text{data}} \ln t(O) - (1 - t_{\text{data}}) \ln(1 - t(O))$$

t_{data} takes 0 or 1 according to true or false

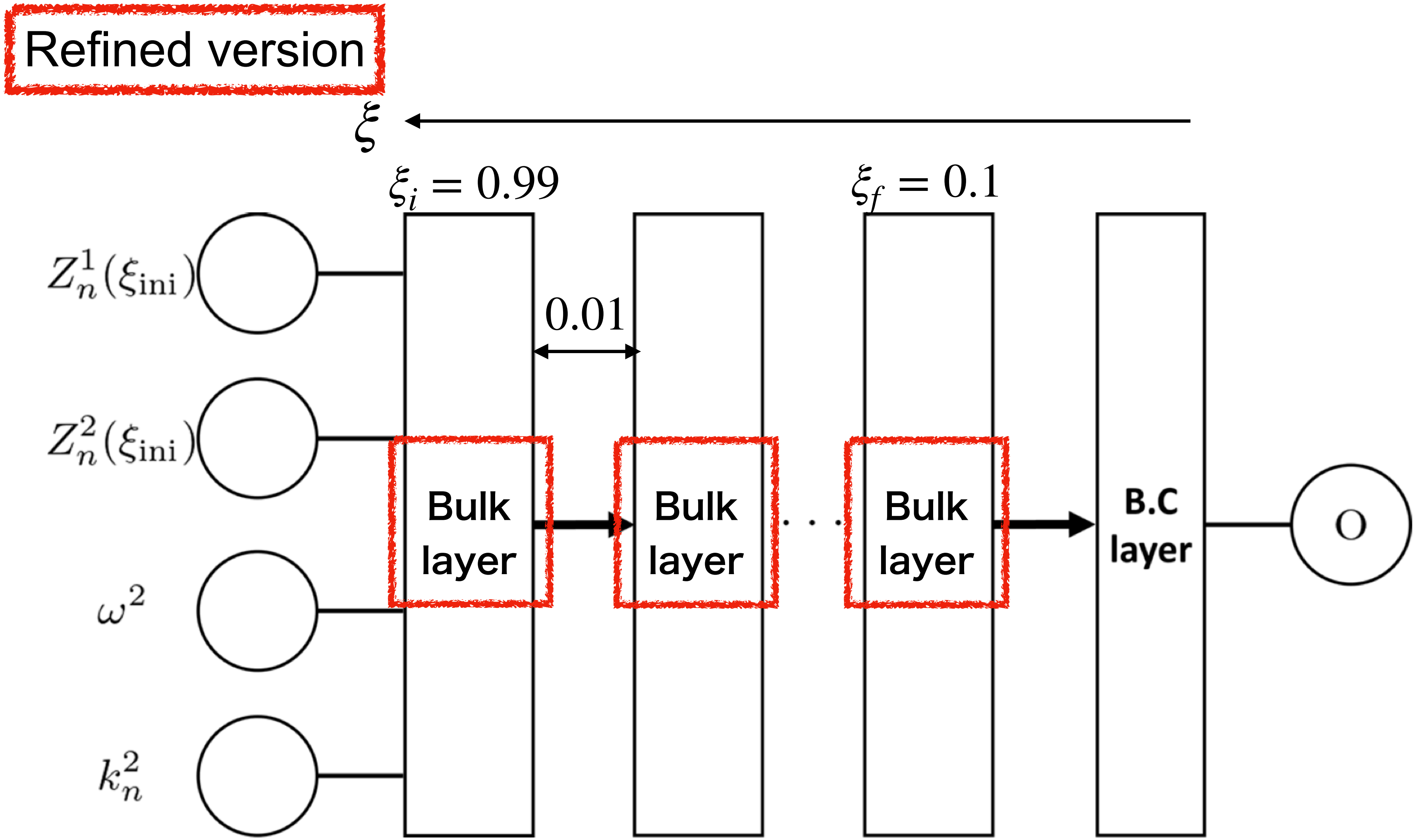
$$t(O) := \frac{1}{2}\left[\tanh(100(O - 0.1)) - \tanh(100(O + 0.1)) + 2\right]$$



$$O = \left| \frac{\xi_f Z^1(\xi_f) + \rho Z^2(\xi_f)}{\sqrt{\xi_f Z^1(\xi_f) + \rho Z^2(\xi_f) + \epsilon}} \right|$$

From Hashimoto, Sugishita, Tanaka, Tomiya (2018)

Solve with binary classification



Data of the linear response (true and false)

Discretized EOM with weight (Ξ, Θ)

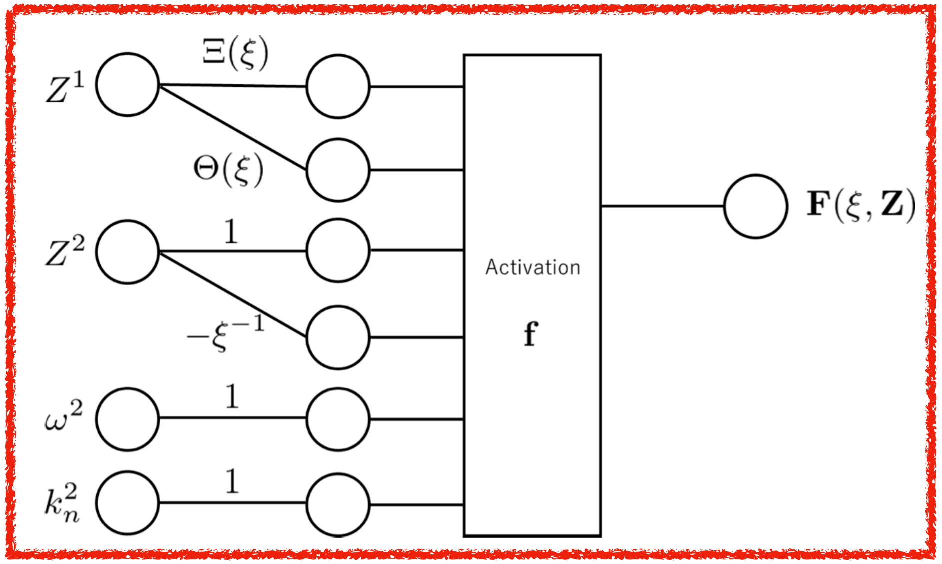
Ask if output matches B.C. with weight ρ

Runge-Kutta layer

EOM

$$Z'(\xi) = \begin{pmatrix} 0 & 1 \\ \omega^2 \Xi(\xi) + k^2 \Theta(\xi) & -1/\xi \end{pmatrix} Z(\xi) =: F(\xi, Z(\xi))$$

Euler method



$$Z(\xi + \Delta\xi) = Z(\xi) + \Delta\xi F(\xi, Z(\xi))$$

Runge-Kutta method

$$Z(\xi + \Delta\xi) = Z(\xi) + \frac{\Delta\xi}{6}\left[F_1 + 2F_2 + 2F_3 + F_4\right]$$

$$F_1 = F(\xi, Z(\xi)), \qquad F_2 = F(\xi + \Delta\xi/2, Z(\xi) + F_1\Delta\xi/2)$$

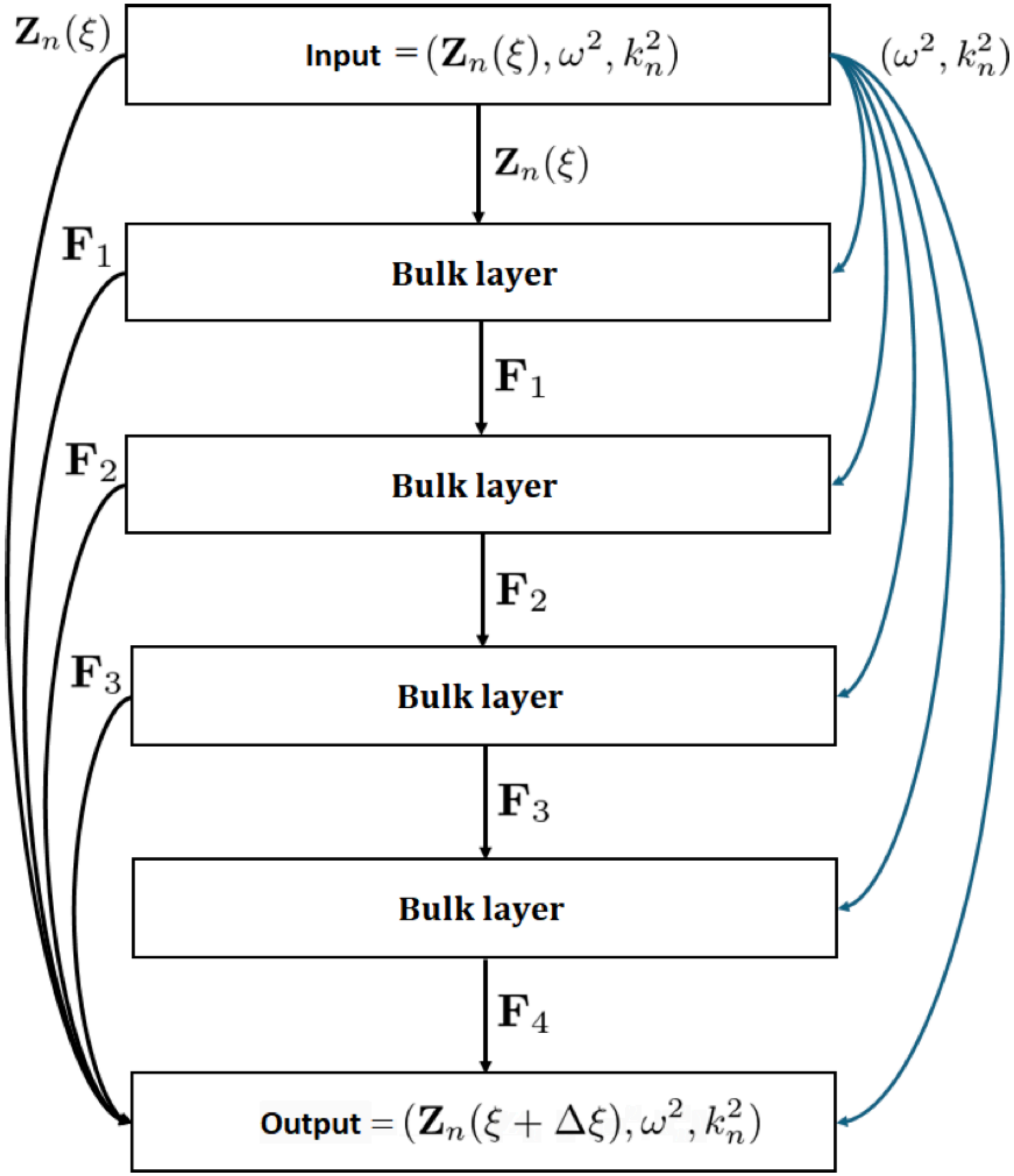
$$F_3 = F(\xi + \Delta\xi/2, Z(\xi) + F_2\Delta\xi/2), \qquad F_4 = F(\xi + \Delta\xi, Z(\xi) + F_3\Delta\xi)$$

Runge-Kutta layer

Runge-Kutta method

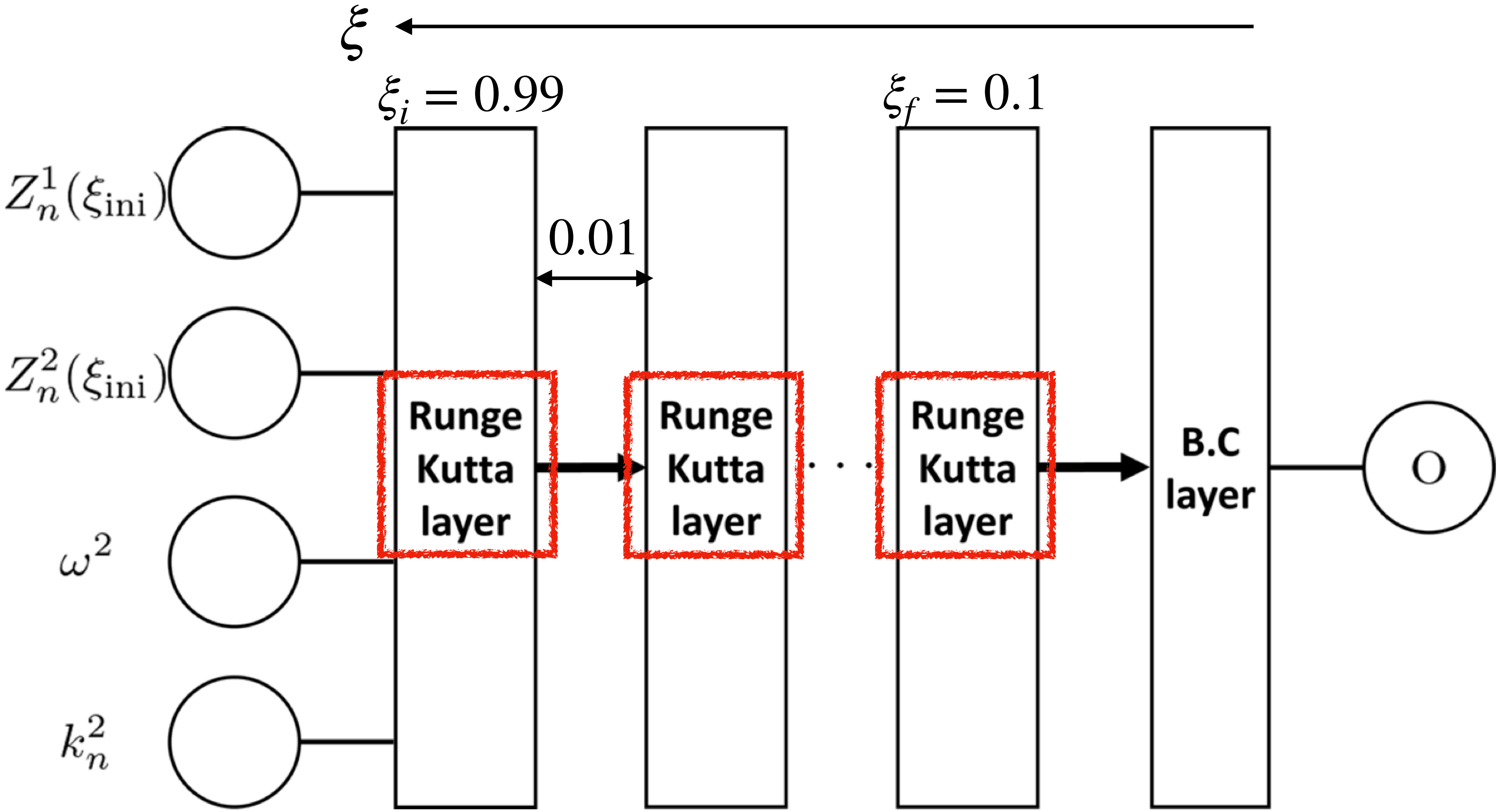
$$Z(\xi+\Delta\xi) = Z(\xi) + \frac{\Delta\xi}{6}\left[F_1 + 2F_2 + 2F_3 + F_4\right]$$

$$F_1 = F(\xi, Z(\xi))$$
$$F_2 = F(\xi + \Delta\xi/2, Z(\xi) + F_1\Delta\xi/2)$$
$$F_3 = F(\xi + \Delta\xi/2, Z(\xi) + F_2\Delta\xi/2)$$
$$F_4 = F(\xi + \Delta\xi, Z(\xi) + F_3\Delta\xi)$$



Solve with binary classification

Refined version



Data of the linear response (true and false)

Discretized EOM with weight (Ξ, Θ)

Ask if output matches B.C. with weight ρ

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The forward problem on BTZ spacetime

BTZ: 3D AdS black hole spacetime (vacuum Einstein solution)

Boundary linear response data computed from the bulk

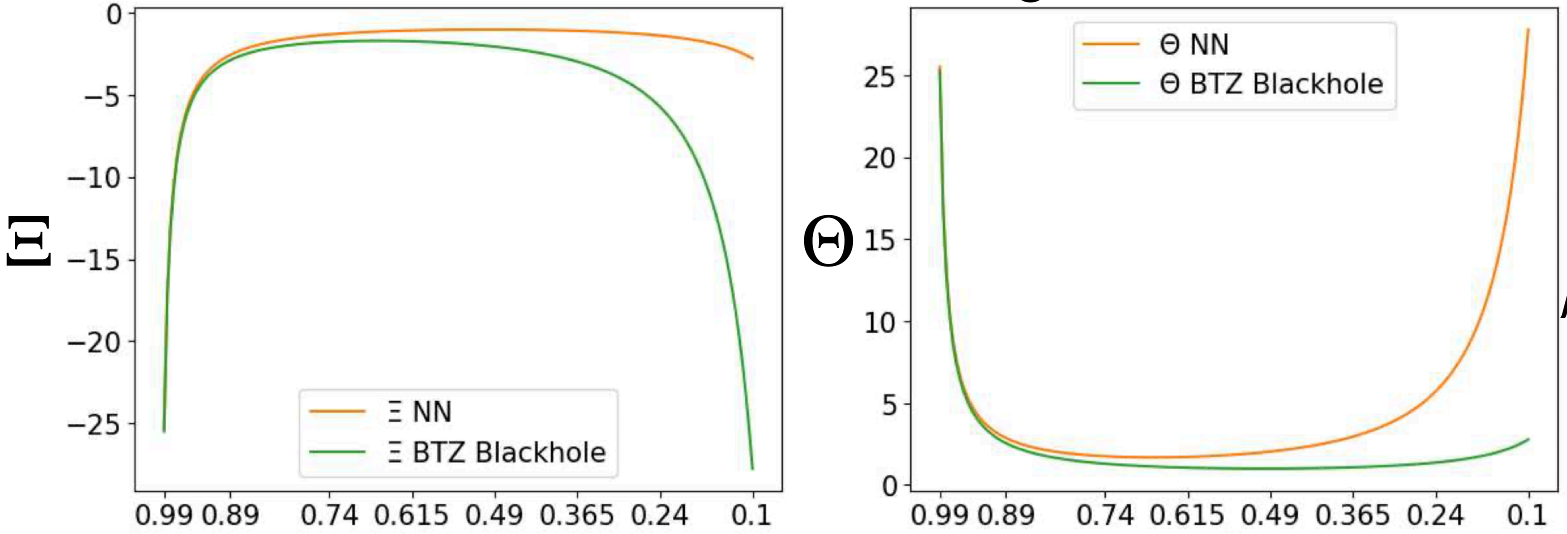
$$\langle O(\omega, k)\rangle_J \propto \frac{\Gamma(1-\frac{iL^2}{2r_h}(\omega-k))\Gamma(1-\frac{iL^2}{2r_h}(\omega+k))}{\Gamma(-\frac{iL^2}{2r_h}(\omega-k))\Gamma(-\frac{iL^2}{2r_h}(\omega-k))} J(\omega, k)$$

Generate true data from this, and also prepare fake data

(with $L = r_h = 1$)

Solving inverse problem by ML

Before learning

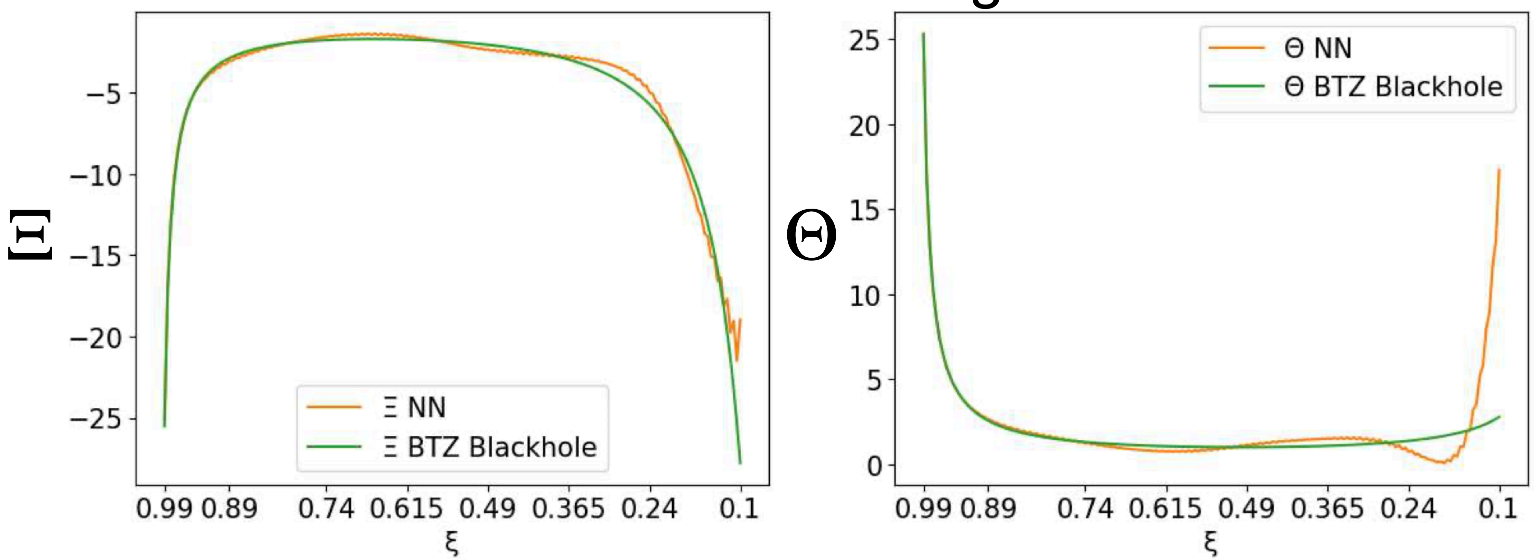


boundary ξ horizon boundary ξ horizon

B.C. layer

$$\rho = 0.50 i\omega - 0.50\,|k|$$

After learning



$$\rho = 0.50 i\omega + 0.01\,|k|$$

Horizon emerges!

Machine-learning emergent spacetime from linear response

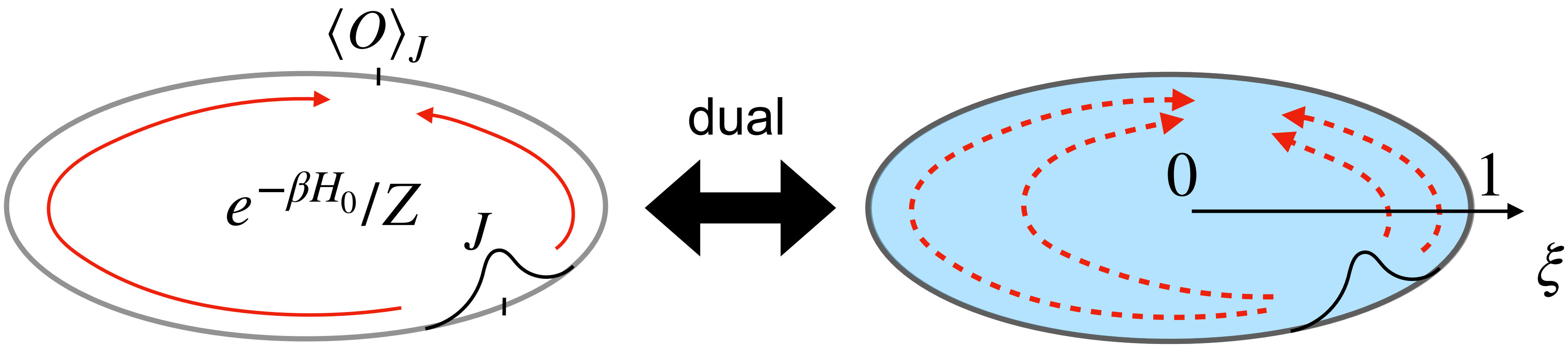
1. The forward problem:
   Boundary linear response from bulk

2. The inverse problem:
   Bulk metric from boundary linear response

3. NN solves the inverse problem

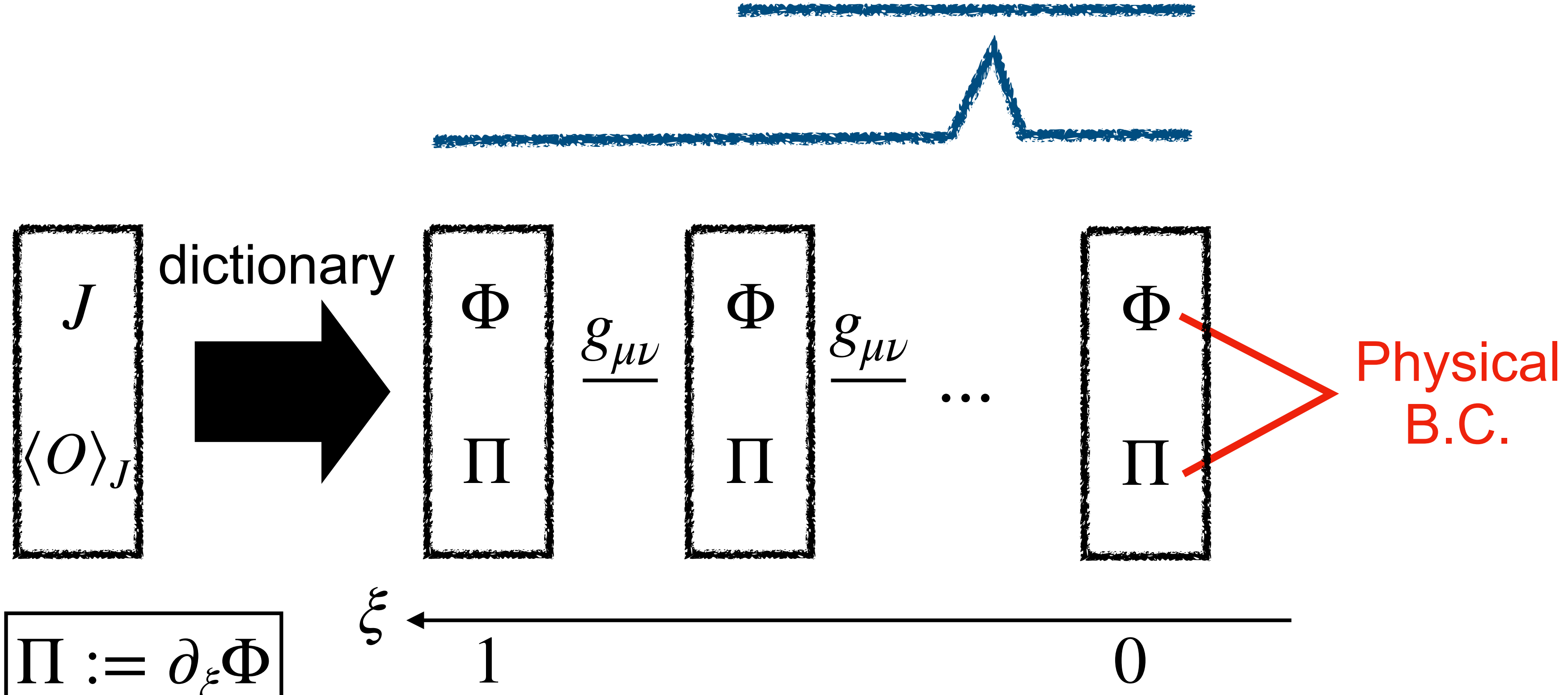
4. Demonstration:
   NN reproduces BTZ metric

Machine-learning $g_{\mu\nu}$ from linear response



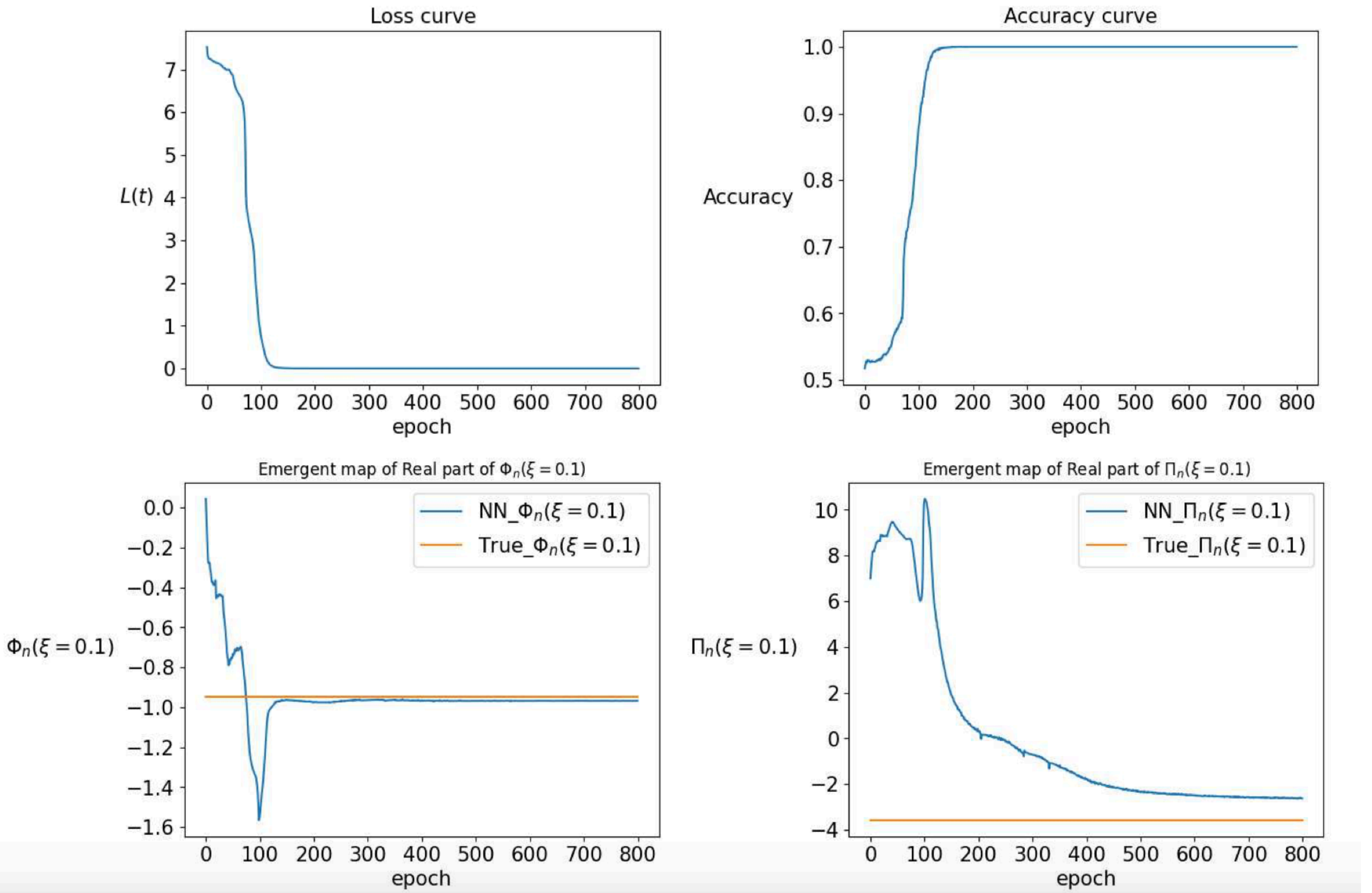
$\dot{\rho} = -i[H(t), \rho]$

$(\Box_g - m^2)\Phi = 0$ with unknown $g_{\mu\nu}$



$\boxed{\Pi := \partial_\xi \Phi}$

Other graphs



Other graphs