Coarse-graining black holes out of equilibrium with boundary observables on time slice

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Self introduction

Name
Daichi Takeda

Status
PhD student (final year from next April)

Likes
Music (youtube ch. Daichi Takeda)

Research topics
Classical solutions in string field theory

Bulk metric reconstruction

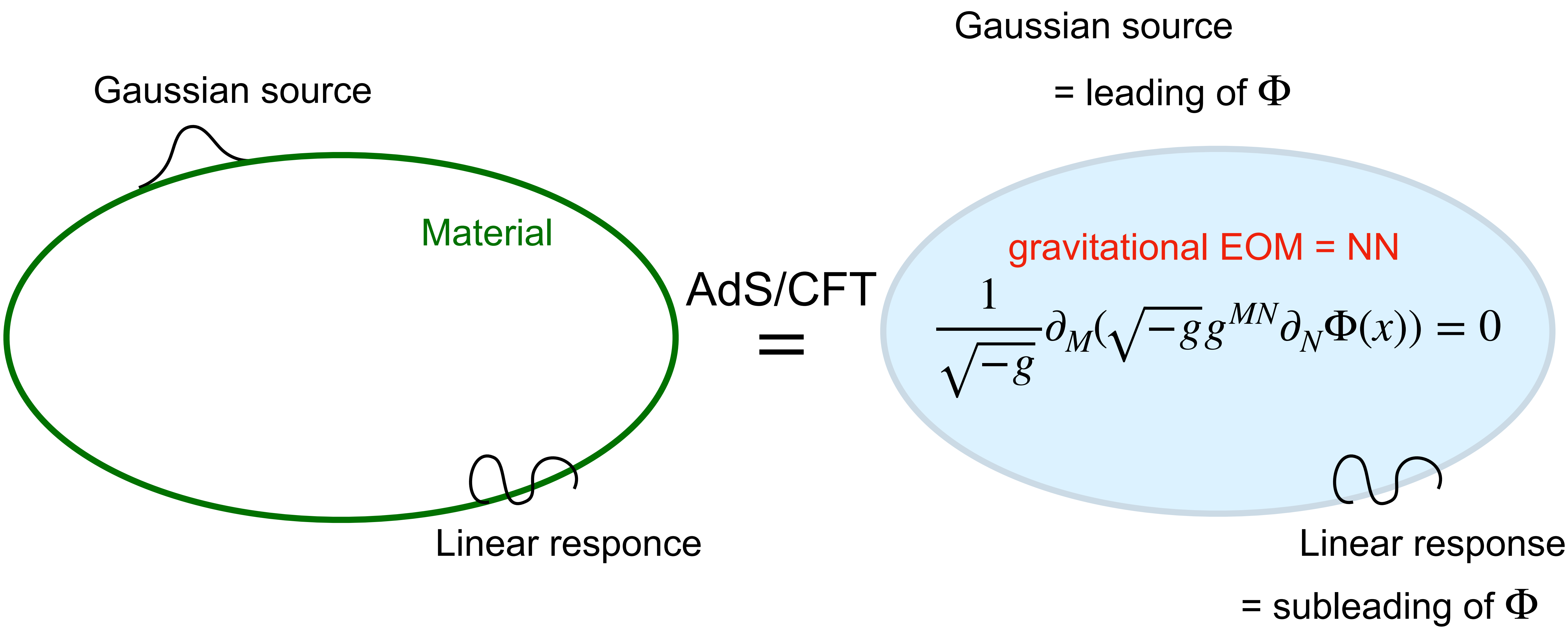
Tabletop quantum gravity experiments

} ← DL involved

Black hole thermodynamics ← Today's main

[Ongoing] Building dual spacetime by DL

Hashimoto, Matsuo, Murata, Ogiwara, DT



Find metric consistent with source and response

From now on, Black holes!

String must constrain BH phys

Origin of spacetime is unknown

Is it string?

On the other hand, BH is thermodynamic

Does it come from statistical mechanics of string?
(Strominger-Vafa 1996)

More messages from string?

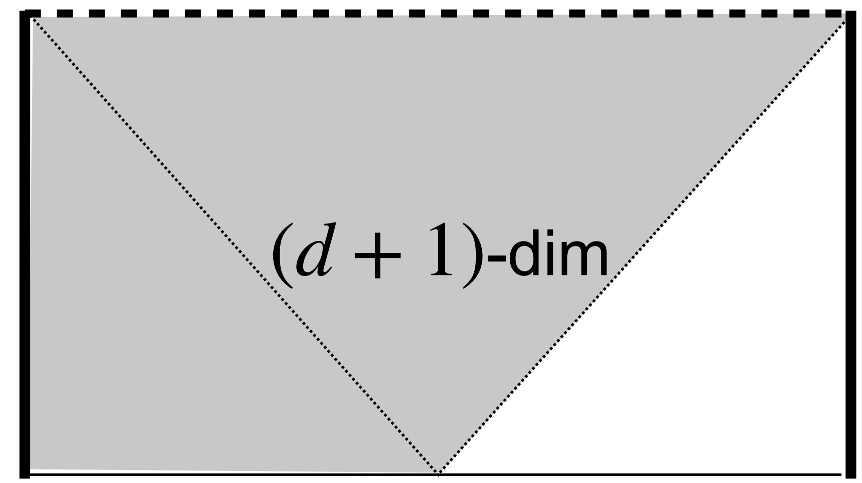
A formulation of non-equilibrium BHT

Unitary evolution of
mixed state

CFT$_d$

AdS/CFT
$=$

Dynamical BH system



Coarse-grained entropy

BH entropy out of equilibrium
(= Bekenstein-Hawking in equilibrium)

a trivial inequality

a nontrivial inequality (2nd law)

1st law in GR

A proposal for BH entropy out of equilibrium obeying 1st and 2nd laws

1. BH thermodynamics and problems

2. 2nd law in CFT is trivial

3. 2nd law written in gravity is nontrivial

4. 1st law is generalized

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0th and 1st laws are successful

Equilibrium thermodynamics (stationary black holes)

0th law: Existence of intensive variables

In BHT, constants over horizon; κ, Ω, ϕ, …

1st law: Energy conservation

In BHT, $\mathrm{d}M = T\,\mathrm{d}S + \Omega\,\mathrm{d}J + \phi\,\mathrm{d}Q + \cdots$

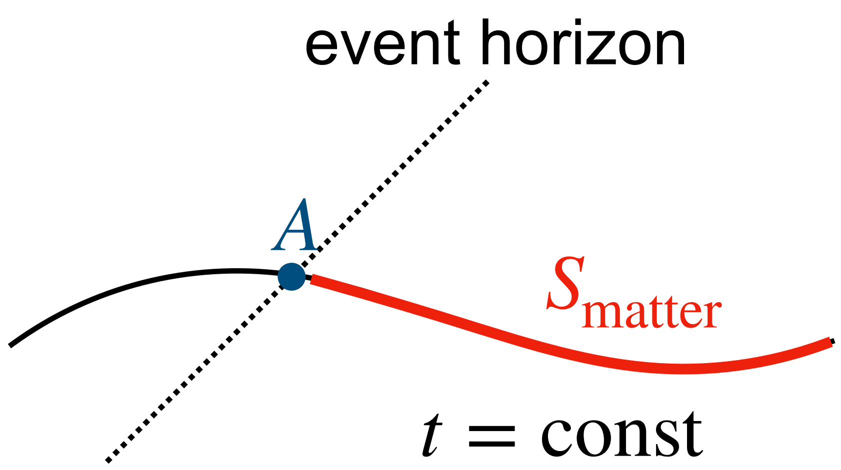
3rd law: $T=0$ cannot be achieved by finite steps

In BHT, this can be broken, but no problem

2nd law has been under debate

2nd law: The entropy at t must not be smaller than that of initial equilibrium state

Generalized entropy?



$$S_{\rm gen} := \frac{A}{4G} + S_{\rm matter}$$

Yet no complete proof for 2nd law

Also, other candidates for entropy proposed

Mainly discussed bottom-up → Let's listen to string

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Coarse-graining is respecting some aspects

Coarse-grained state $\rho_{\mathrm{cg},t}$

ρ_t: target state $\{O_I(\vec{x})\}$: operator set

$$\rho_{\mathrm{ref},t} := \mathrm{argmax}_\rho \left(-\mathrm{Tr}\rho \ln \rho\right)$$

↑

under $\mathrm{Tr}(\rho\, O_I(\vec{x})) = \mathrm{Tr}(\rho_t\, O_I(\vec{x}))$

$$\rho_{\mathrm{cg},t} \propto \exp\left[\int \mathrm{d}^{d-1}\vec{x}\, \lambda_t^I(\vec{x}) O_I(\vec{x})\right]$$

λ_t : Lagrange multipliers

Coarse-grained entropy S_t

$$S_t := -\,\mathrm{Tr}\rho_{\mathrm{cg},t} \ln \rho_{\mathrm{cg},t} = -\int \mathrm{d}^{d-1}\vec{x}\, \lambda_t^I(\vec{x})\, \mathrm{Tr}(\rho_t\, O_I(\vec{x})) + \ln Z_t$$

2nd law holds trivially

Setup

$$\rho_0 \propto \exp\left[\int \mathrm{d}^{d-1}\vec{x}\, \lambda_0^I(\vec{x}) O_I(\vec{x})\right]$$

$$H(t) = H - \int \mathrm{d}^{d-1}\vec{x}\, j^I(t,\vec{x}) O_I(\vec{x})$$

$$\rho_t$$

Positivity of relative entropy

$$\mathrm{Tr}\,\rho_t \left(\ln \rho_t - \ln \rho_{\mathrm{cg},t}\right) \geq 0 \quad \Longleftrightarrow \quad \text{2nd law} \quad S_t \geq S_0$$

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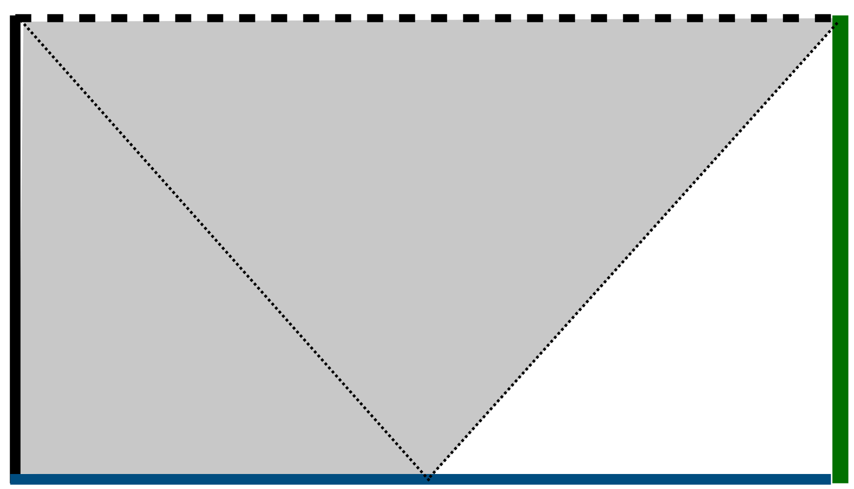
Coarse-graining BH out of equilibrium

Setup

Dynamical BH system



CFT Operators to be respected

$$H \qquad \{O_I(\vec{x})\}$$

On-shell Lorentzian action

GKPW says

$$\langle H \rangle_t = (\text{ADM mass})\big|_t \qquad \langle O_I(\vec{x}) \rangle_t = \frac{\delta}{\delta j^I(t,\vec{x})} I_{\text{grav}}[j]$$

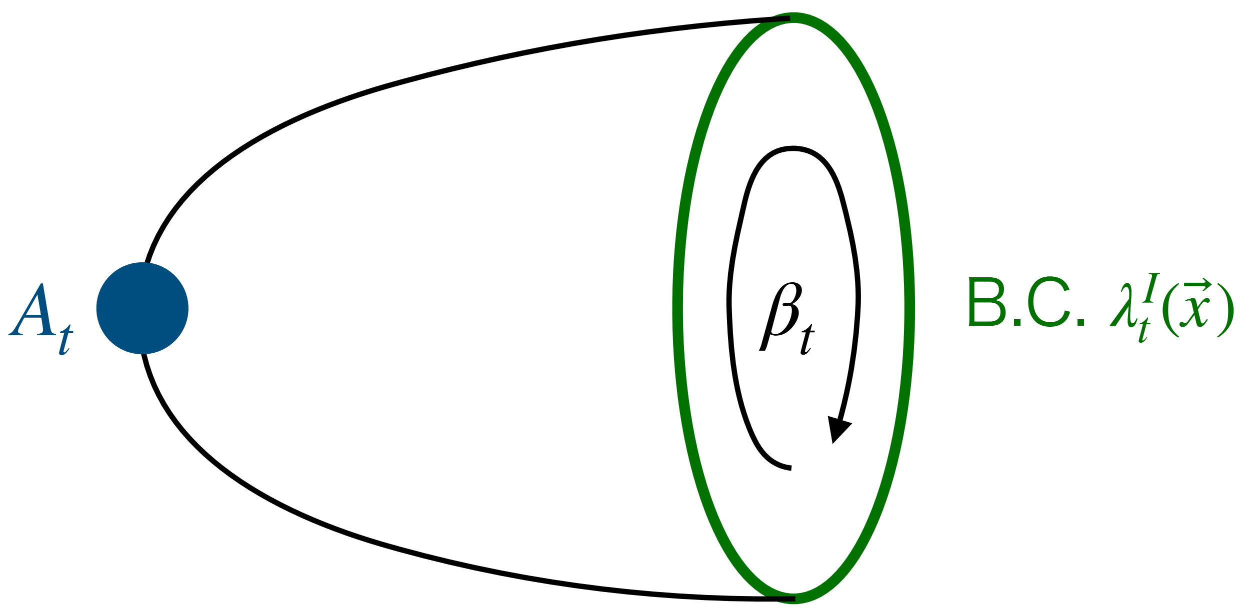
Entropy is horizon area, but in Euclidean

Partition function of coarse-grained state in CFT

$$Z[\underline{\beta_t, \lambda_t}] \propto \mathrm{Tr}\exp\left[-\beta_t H + \int \mathrm{d}^{d-1}\vec{x}\,\lambda_t^I(\vec{x})O_I(\vec{x})\right] = \oint \mathscr{D}\phi\, e^{-I_{\mathrm{CFT}}^{(\mathrm{E})}[\phi;\beta_t,\lambda_t]}$$

Multipliers to be determined via
$\mathrm{Tr}(\rho_{\mathrm{cg},t}\, O_I(\vec{x})) = \mathrm{Tr}(\rho_t\, O_I(\vec{x}))$

Dual solution is Euclidean (E)



$$S_t = \frac{A_t}{4G}$$

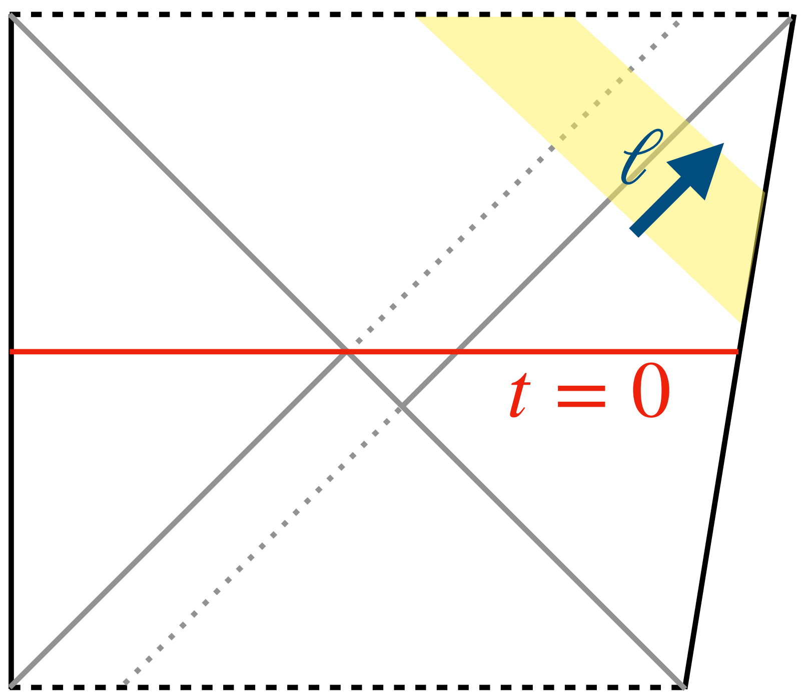
β_t, λ_t are determined so that

$$(\text{ADM mass})\big|_t = (\text{ADM mass in (E)}) \qquad \frac{\delta}{\delta j^I(t,\vec{x})} I_{\mathrm{grav}}[j] = -\frac{\delta}{\delta \lambda_t^I(\vec{x})} I_{\mathrm{grav}}^{(E)}[\beta_t, \lambda_t]$$

2nd law holds nontrivially

AdS/CFT requires $S_t \geq S_0$

Ex) AdS-Vaidya



$S_t \geq S_0$ does not hold always

It holds if $T_{\ell\ell} \geq 0$

The same is confirmed also in other Vaidya models

Constraints by string

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First law is generalized

Entropy S_t and free energy $F_t := \beta_t^{-1} I_{\rm grav}^{(\rm E)}[\beta_t, \lambda_t]$

$$S_t = -\beta_t F_t + \beta_t \langle H \rangle_t - \int \mathrm{d}^{d-1}\vec{x}\, \lambda_t^I(\vec{x}) \langle O_I(\vec{x}) \rangle_t$$

Considering variation of $I_{\rm grav}^{(\rm E)}[\beta_t, \lambda_t]$ and using EOM,

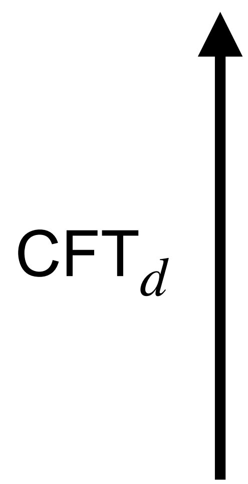
$$\dot{S}_t = \beta_t \frac{\mathrm{d}}{\mathrm{d}t} \langle H \rangle_t - \int \mathrm{d}^{d-1}\vec{x}\, \lambda_t^I(\vec{x}) \frac{\mathrm{d}}{\mathrm{d}t} \langle O_I(\vec{x}) \rangle_t$$

AdS/CFT is not used. Einstein theory only.

A formulation of non-equilibrium BHT

Unitary evolution of mixed state

Dynamical BH system



AdS/CFT

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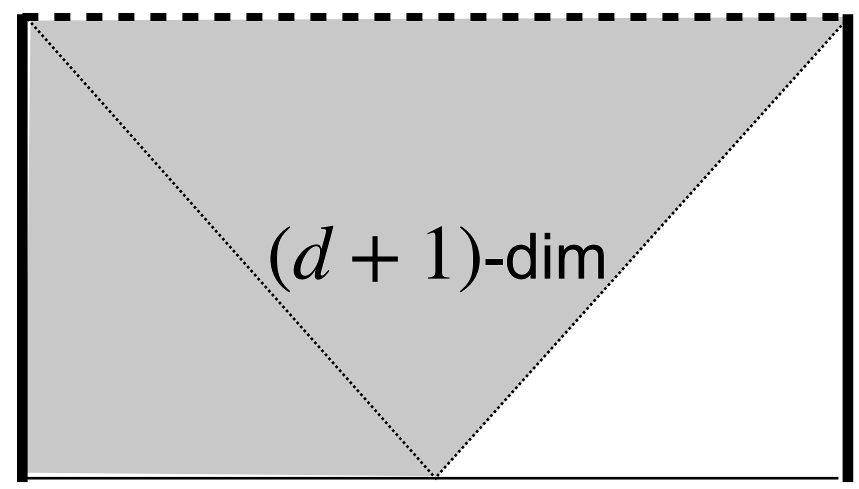

Coarse-grained entropy

Area of Euclidean BH

a trivial inequality

Area inequality

generalized 1st law in GR