Interior Product, Lie derivative and Wilson line in the *KBc* sector of Open String Field Theory

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Abstract

The similarities between Witten's open SFT and Chern-Simons theory

$\Downarrow$

Any other correspondence?

$\Downarrow$

Introduction of the notion of the manifold to *KBc* sector

$\Downarrow$

- Classical solutions on the manifold
- Wilson lines on the manifold

Contents

- Witten's open string field theory
- Classical solutions
- *KBc* manifold and Wilson line
- Summary

Contents

- Witten's open string field theory (3)
- Classical solutions
- *KBc* manifold and Wilson line
- Conclusion and outlook

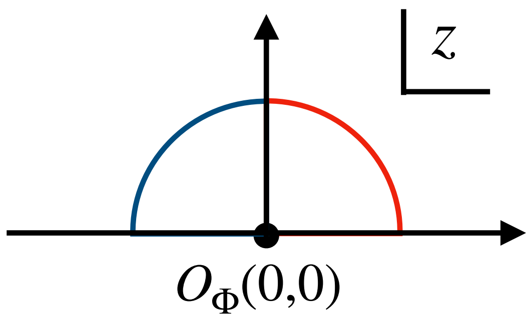
Open string field

Dynamical variable Ψ

$$\hat{X}^{\mu}(0,\sigma)\,|X\rangle = X^{\mu}(\sigma)\,|X\rangle$$

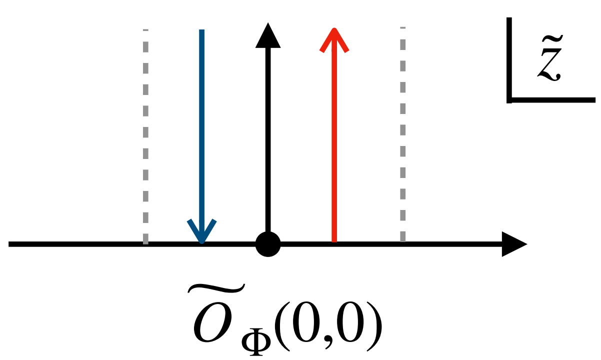
$$\Phi(X(\sigma)) = \langle X\,|\,\Phi\rangle$$ ← A state of world-sheet BCFT

State-operator correspondence $\quad |\,\Phi\rangle = O_{\Phi}(0,0)\,|\,0\rangle$



$$\boxed{\tilde{z} = \frac{2}{\pi}\arctan z}$$

sliver frame



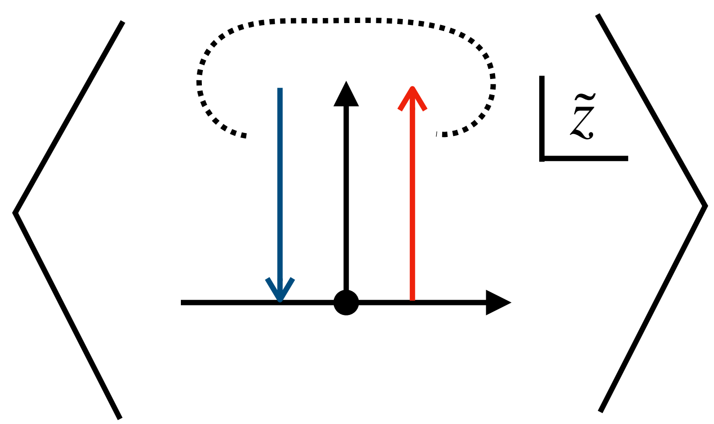
Ψ: A composite operator in sliver frame

ghost number 1

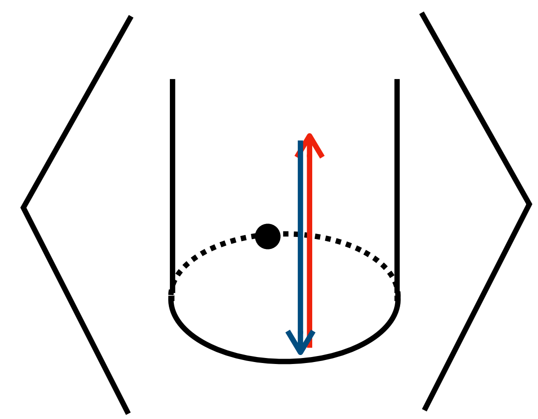
One string vertex (Witten's integral)

One string vertex

$$\int A = $$



$$=$$



ghost number 3

Correlation function with identification of left- and right-half string

Witten's action

Witten's action

BRST operator

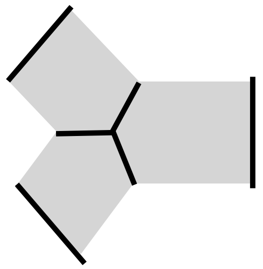
$$S = -\frac{1}{g^2}\int \underbrace{\left(\frac{1}{2}\Psi Q_{\mathrm{B}}\Psi + \frac{1}{3}\Psi^3\right)}_{\text{ghost number 3}}$$

Propagator

$$\frac{1}{2}\Psi Q_{\mathrm{B}}\Psi \quad \Rightarrow$$

Vertex

$$\frac{1}{3}\Psi^3 \quad \Rightarrow$$



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- Witten's open string field theory
- Classical solutions (2)
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Classical solutions

Witten's action

$$S = -\frac{1}{g^2}\int\left(\frac{1}{2}\Psi Q_{\mathrm{B}}\Psi + \frac{1}{3}\Psi^3\right)$$

$\Rightarrow$

EOM

$$Q_{\mathrm{B}}\Psi + \Psi^2 = 0$$

Finding a classical solution $=$ Going to another perturbative vacuum based on the corresponding BCFT

ex) Creation and annihilation of D branes

KBc sector

The definitions of K, B and c

$$K := \int_{-i\infty}^{i\infty} \frac{d\tilde{z}}{2\pi i} T(\tilde{z}), \quad B := \int_{-i\infty}^{i\infty} \frac{d\tilde{z}}{2\pi i} b(\tilde{z}), \quad c := c^{\tilde{z}}(0) = \frac{2}{\pi} c^{z}(0)$$

(in sliver frame)

***KBc* algebra**

$$\left.\begin{aligned} &[K, B] = B^2 = c^2 = 0, \quad \{B, c\} = 1, \quad ([K, c] = -\partial c) \\ &Q_B K = 0, \quad Q_B B = K, \quad Q_B c = cKc \end{aligned}\right\} \text{Closed}$$

Classical solutions have been found in *KBc* sector: $\Psi = F(K, B, c)$
$\rightarrow$ Universal solution

EOM

$$Q_B \Psi + \Psi^2 = 0$$

Contents

- Witten's open string field theory
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The similarities between SFT and CS theory

Witten's action

$$S = -\frac{1}{g^2}\int\left(\frac{1}{2}\Psi Q_{\mathrm{B}}\Psi + \frac{1}{3}\Psi^3\right)$$

Chern-Simons action

$$S_{CS} \sim \int_M\left(\frac{1}{2}A\mathrm{d}A + \frac{1}{3}A^3\right)$$

Correspondence

$$Q_{\mathrm{B}} \quad \leftrightarrow \quad \mathrm{d}$$

$$\int \quad \leftrightarrow \quad \int_M$$

ghost $\leftrightarrow$ form

$$\Psi \to V^{-1}(Q_{\mathrm{B}} + \Psi)V \quad \leftrightarrow \quad A \to g^{-1}(\mathrm{d} + A)g$$

gauge transformation

Any other correspondence? $\Rightarrow$ Yes (restricting to *KBc* sector)

[H.Hata, DT (2021)]

KBc interior product 1/2

Finding the interior product I in *KBc* sector

Assumptions

- I has ghost number -1 ← ghost $\leftrightarrow$ form

- I holds *KBc* algebra ← The consistency with KBc algebra

  ex) $I(\{B,c\}) = I(1)$ for $\{B,c\} = 1$

- $I(AB) = (IA)B + (-1)^{|A|}A(IB)$ ← Same as the ordinary Interior product

KBc interior product 2/2

Then, I is characterized by a two-component function of K, $X = (X_1(K), X_2(K))$

$$\boxed{I_X K = iBX_1, \quad I_X B = 0, \quad I_X c = \frac{X_2}{K} + \left[\frac{X_2}{K}, Bc\right]}$$

$X = (X_1(K), X_2(K))$: *KBc* tangent vector

The same relations as the ordinary one

$$I_X^2 = 0, \quad \{I_X, I_Y\} = 0, \quad I_{\alpha X + \beta Y} = \alpha I_X + \beta I_Y$$

KBc Lie derivative

Lie derivative | The ordinary one

$L_X := -i\{Q_\mathrm{B}, I_X\} \longleftarrow \mathscr{L}_X = \{\mathrm{d}, I_X\}$

The same relations as the ordinary one with $\mathrm{d} \leftrightarrow Q_\mathrm{B}$

$$[L_X, Q_\mathrm{B}] = 0, \quad [L_X, I_Y] = [I_X, L_Y], \quad L_X(AB) = (L_X A)B + A L_X B, \quad L_{\alpha X+\beta Y} = \alpha L_X + \beta L_Y$$

The other expected formulas $\quad [L_X, I_Y] = I_{[X,Y]},\ [L_X, L_Y] = L_{[X,Y]}$

These hold by the replacement of $[X, Y]$ with

$$[X, Y] := (X_1 K Y_1' - Y_1 K X_1', X_1 K Y_2' - Y_1 K X_2')$$

Lie bracket!

$Y_1' = Y_1'(K)$

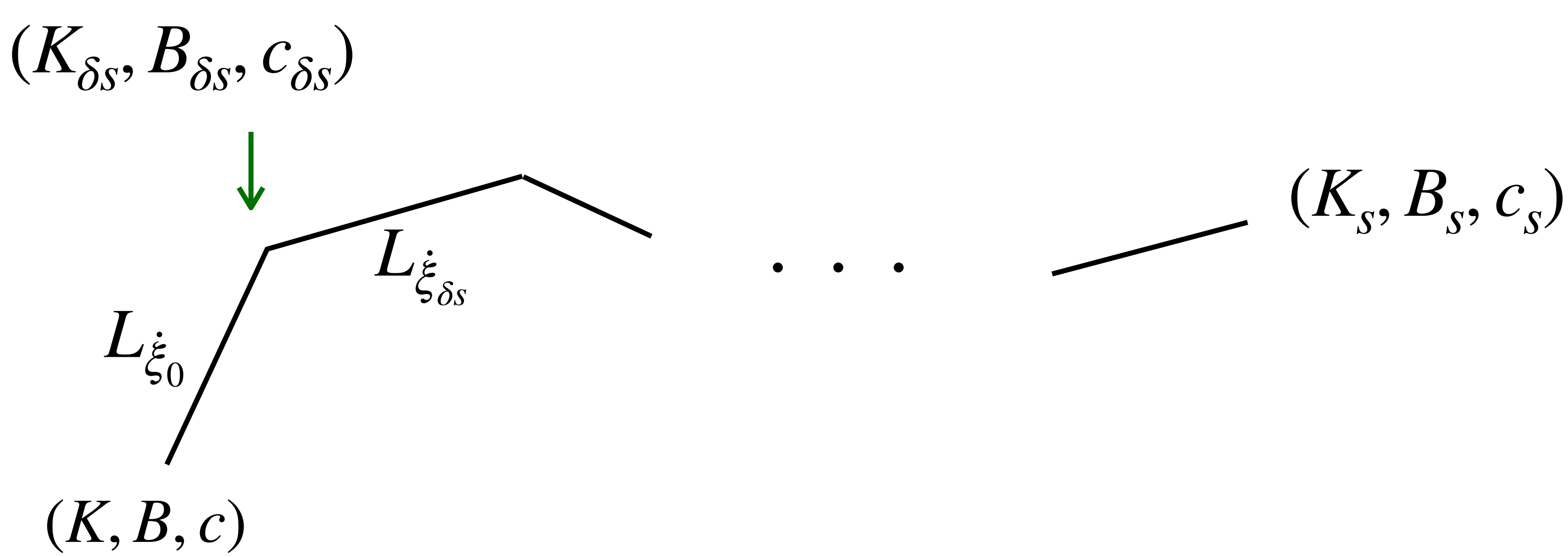
KBc manifold 1/2

The new triad $(1+L_X)(K,B,c)$ also forms *KBc* algebra !

Therefore, using a function $\xi(s)=(\xi_1(s,K),\xi_2(s,K))$ and solving

$$\frac{\mathrm{d}}{\mathrm{d}s}(K_s,B_s,c_s)=L^{(s)}_{\dot\xi(s)}(K_s,B_s,c_s),\quad (K_0,B_0,c_0)=(K,B,c)$$

give various *KBc* algebra.



The solution only depend on the end point

$$\boxed{K_s=e^{\xi_1(s,K)}K,\quad B_s=e^{\xi_1(s,K)}B,\quad c_s=e^{-i\xi_2(s,K)}ce^{-\xi_1(s,K)}Bce^{i\xi_2(s,K)}}$$

KBc manifold 2/2

Many different triads of *KBc* are obtained:

$$K(\xi) = e^{\xi_1(K)}K, \quad B(\xi) = e^{\xi_1(K)}B, \quad c(\xi) = e^{-i\xi_2(K)}ce^{-\xi_1(K)}Bce^{i\xi_2(K)}$$

$$\boxed{\xi = (\xi_1(K), \xi_2(K))}$$

***KBc* manifold**

- Points $\longrightarrow$ different *KBc* triads
- Coordinate $\longrightarrow$ $\xi = (\xi_1(K), \xi_2(K))$

$Q_{\mathrm{B}},\ I_X,\ L_X$ are generalized onto *KBc* manifold $\longrightarrow$ $Q_{\mathrm{B}},\ \underline{I_X^{(\xi)},\ L_X^{(\xi)}}$

regarding $(K(\xi), B(\xi), c(\xi))$

as the fundamental triad

Classical solutions on *KBc* manifold

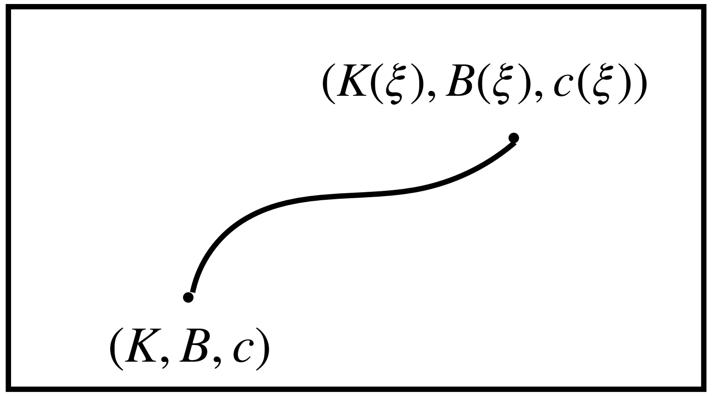
A classical solution Ψ is extended to the quantity on *KBc* manifold:

$$\Psi(\xi) := \Psi\Big|_{(K,B,c)\to((K(\xi),B(\xi),c(\xi))}$$

If Ψ is a classical solution, then $\Psi(\xi)$ is again a classical solution.

However, $\Psi(\xi)$ is gauge-equivalent to Ψ, If $(K(\xi), B(\xi), c(\xi))$ can be connected to (K, B, c) with a continuous curve.

KBc manifold



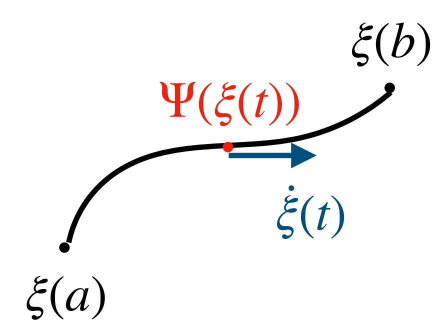
Wilson line on *KBc* manifold

Wilson line in CS theory

$$W_C = \mathrm{P}\exp\left[\int_C A_\mu(x)\mathrm{d}x^\mu\right] = \mathrm{P}\exp\left[\int_a^b \mathrm{d}t\ \underbrace{i_{\dot{x}(t)}}_{\text{the interior product on } M_3} A(x(t))\right]$$

By analogy with CS theory…

$$W_C = \mathrm{P}\exp\left[i\int_a^b \mathrm{d}t\ \underbrace{I^{(\xi(t))}_{\dot{\xi}(t)}\Psi(\xi(t))}_{\text{ghost number 0}}\right]$$



Some similar properties to the ordinary Wilson line hold for the *KBc* version.

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Summary

The construction of *KBc* manifold (top-down)

1. $K(\xi) = e^{\xi^1} K, \quad B(\xi) = e^{\xi^1} B, \quad c(\xi) = e^{-i\xi^2} c e^{-\xi^1} B c e^{i\xi^2}$ form *KBc* algebra.

2. *KBc* manifold

$$\text{point} : (K(\xi), B(\xi), c(\xi)), \qquad \text{coordinate} : \xi = (\xi_1(K), \xi_2(K))$$

3. Tangent vector X, interior product $I_X^{(\xi)}$ and Lie derivative $L_X^{(\xi)}$ can be properly defined.

Conclusion

- Classical solutions are extended onto *KBc* manifold.
- Wilson lines are defined on *KBc* manifold.

Outlook

- **The physical meaning of *KBc* manifold**

  Can $(K(\xi), B(\xi), c(\xi))$ be regarded as (K, B, c) in another BCFT?

  $\rightarrow$ The relation between *KBc* manifold and BCFT

- **Wilson loop**

  Wilson loop cannot be naively defined by analogy with CS theory.

  Wilson loop requires an operator which has the cyclic property.

  One string vertex $\int$ does, but needs ghost number 3.